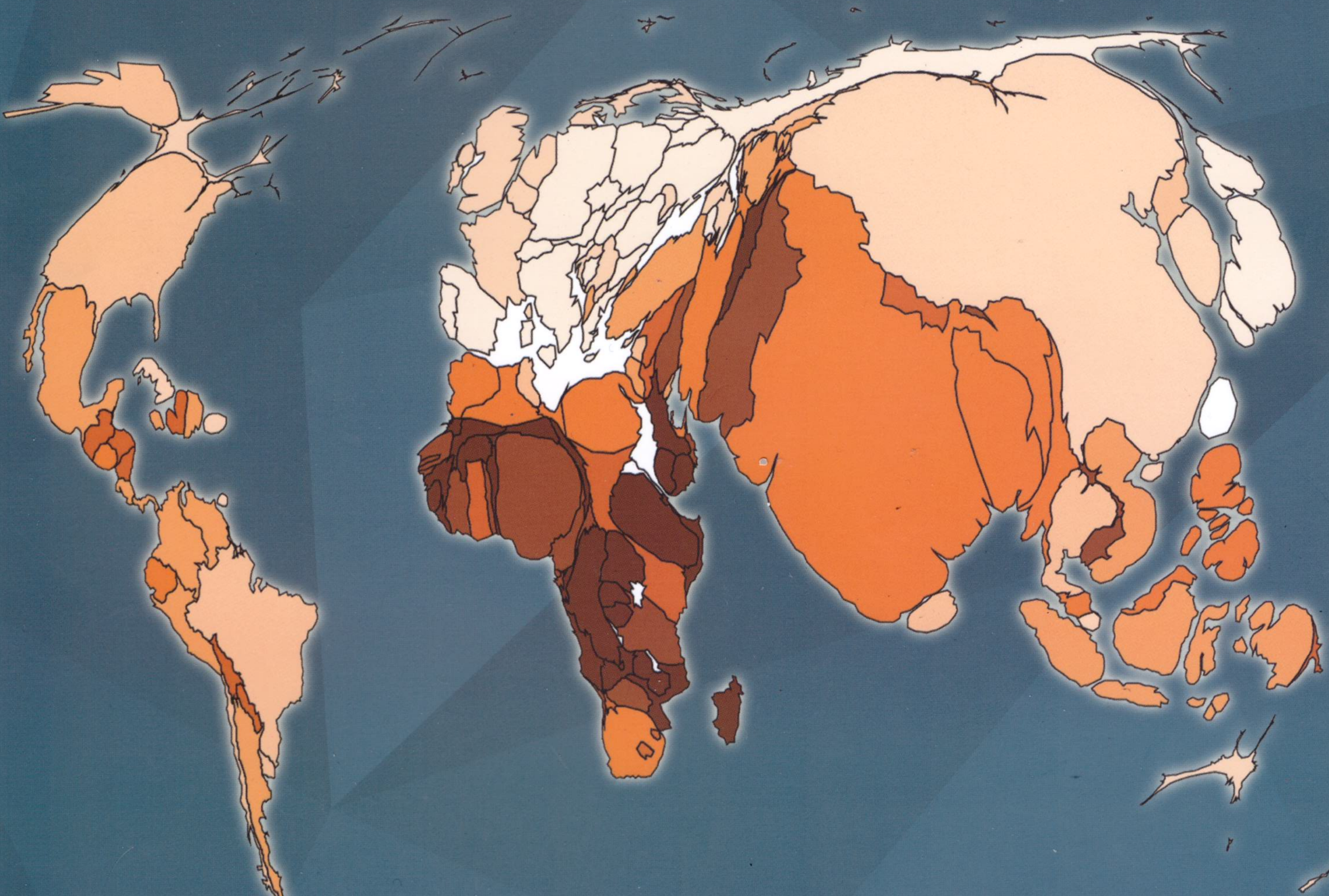
Visualisation In Noneuclidean Metrics



Sabir M. Gusein-Zade, Vladimir S. Tikunov

The publishing of the book is funded by the National Basic Research Program of China (2015CB954103)

Sabir M. Gusein-Zade is a Professor of the Department of higher geometry and topology of the Moscow State Lomonosov University. He published more than 180 papers in pure and applied mathematics. In geography his interests concentrate around models of distribution of population and construction of non-traditional cartographic images. He is a member of the Editorial Boards of 8 mathematical journals; in particular he is the Editor-in-chief of the Moscow Mathematical Journal.

Vladimir S. Tikunov is Professor, Head of the Integrated Mapping Laboratory, Geography Faculty of the Moscow State Lomonosov University, and Director of the Center of the World Data System for Geography. He published over 500 scientific works, including 14 monographs, texbooks, and manuals in 28 countries of the world and in 14 languages. He has led a number of Russian and international projects. His work has been used in many thematic maps and atlases: National Atlas of Russia (editor-in-chief of Vol. 3), Environmental Atlas of Russia, Atlas of Socio-Economic Development of Russia, Atlas of the Khanty-Mansi Autonomous District Yugra, etc. He is a recipient of the D.N. Anuchin Award for his work in mathematical cartographic modeling. He is also a recipient of the Award in Science and Technology of the Russian Federation Government for the development of environmental and natural-resources atlases of Russia. He was Vice-President and, currently, is Chairman of the commission of the International Cartographic Association and a member of the Commission on Geographic Information Science of the International Geographical Union. He has been a member of the Editorial Boards of 9 Russian and international journals. He lectured at a number of national and international universities. He has been organizing, beginning in 1994, annual international conferences InterCarto-InterGIS "Sustainable Development of Territories: GIS Theory and Practical Experience" that take place both in Russia and abroad.

Introduction

Development of a number of Earth sciences, connected with the spatial-temporal analysis, presupposes not only an improvement of methods of representation of spatial-coordinated phenomena, but also demonstration of their relations and connections with other phenomena, especially in cases when we are analyzing them as systems. Often it is necessary to examine changing over the space characteristics of several phenomena at once. It is much convenient to carry out such an analysis in the case if one of the characteristics is uniformly distributed over the territory and we are regarding all other characteristics against this one as the background. Of cause such a situation is very rear. It arises the idea to create it artificially. For this it is possible to transform the image of the phenomenon taken as the base from usual Euclidean metric of the space into a conditional thematic "space" of the uniformed phenomenon. Under the term "transformation" we understand a transition from the ordinary cartographic image, usually based on the topographic metric of the Earth surface, to another image, based on a metric connected with the phenomenon under consideration. Geographers express growing interest to such transformed images which are called anamorphosises. In other words anamorphosises can be defined as graphical images obtained from the traditional maps, the scale of which is not constant and varies depending on values of some indices, on which they are based.

In English speaking countries instead of anamorphosises terms cartograms, transformed maps, pseudo-cartograms, cartograms, topological cartograms and so on are used. We prefer to use the term anamorphosis and to call an anamorphation (from the greek word anamorphoo) the process of their creation. It seems that this term reflects the essence of the process, connected with the change of image proportions, more precisely. Besides that let us emphasize that this term is spread in a number of countries, first of all in the East Europe. In the Russian scientific language the word "anamorphosis" also has been used for a rather long time. Thus as far back as well-known Russian linguist V.I.Dal' (1881) defined anamorphosis as a hideous but regularly deformed picture, which can be seen in a cut or curved mirror. By the way this definition coincides with one of the methods of creation of anamorphosises, which is used till now. However according to the contemporary concept of anamorphotic images the term "hideous" is hardly in its place. A transformation of cartographic images is produced for theoretical and practical purposes and serves as a tool of the spatial analysis.

Anamorphated images differ from carotids and from well-known mental maps (Gould,White, 1974). Cartoids are abstract graphic images for compilation of which real spatial relations are not important, but there are shown some substantial characteristics: the main essence of phenomena, regularities in their allocation, in their development and in the reasons defining them. Examples of cartoids are: "the ideal continent", "typical relief forms", "inversion carotid of the population system in Africa" (after S.V.Rogachev), reasons for rise of Moscow in the Russian state (after Yu.G.Saushkin and B.B.Rodoman), polarized landscape (after B.B.Rodoman; Fig. 1) etc.

Mental images are graphic representations of ideas about spatial objects formed in human brains. They have been created by all of us when we drew schemes explaining, for example, how is it possible to find the desired place in a city. One can average such representations and obtain a collective mental image. A number of examples like characterization of different places of Los Angeles from the point of view of representatives of middle white, afro-american and hispanospeaking population, ideas of Londoners about the North or the image of the world as it is seen from Van Hornsville village, USA (with such "provincial" centres as New York, London or Moscow) can be found

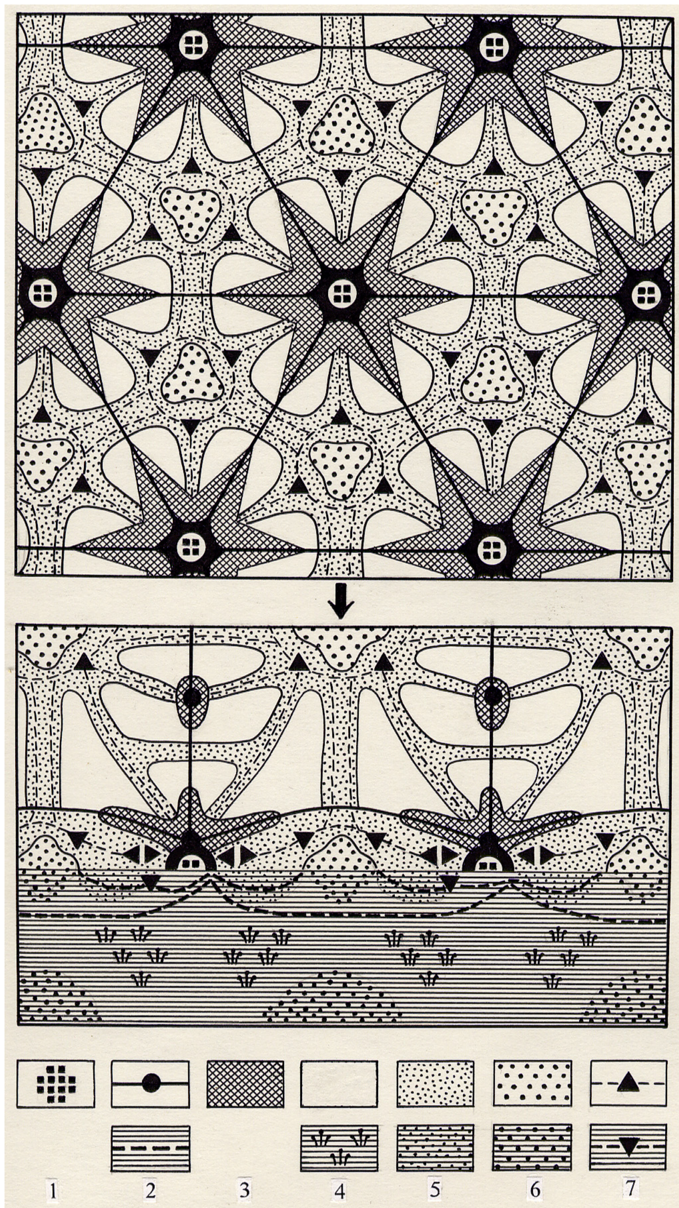


Fig. 1: Net polarised landscape on land and sea. Functional zones and ways of communication: A — for a homogeneous plain inside a continent, B — for a seaside. In the upper raw of the legend for land, in the lower one for sea. 1 – urban historical-architectural reservations; 2 – public service and utilitarian ways of communications; 3 – permanent urban residences and manufacturing industry; 4 – agriculture of high and medium intensity; 5 – natural meadows, pastures, forest industry, hunting, suburban recreational parks; 6 – natural preservations; 7 – recreational residences and touristic roads (after B.B.Rodoman, 1974).

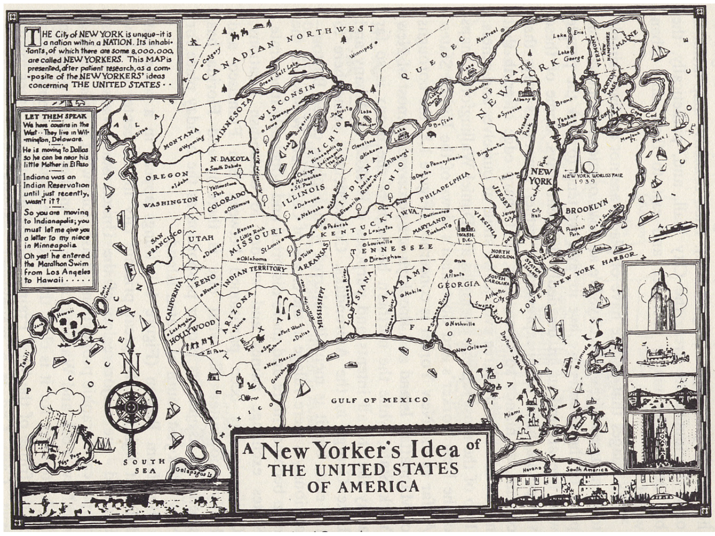


Fig. 2: USA from the New-Yorker's viewpoint

in [P.Gold, R.White, 1974]. Let us recall as well evaluation of Baltic beaches by residents of the North and South Europe, of sizes of lakes around Esbu city (Sweden) from the point of view of students etc. As an illustration let us show well-known images of the USA territory from the point of view of residents of New York and of Boston: Fig. 2 and 3 (after D.Vallingford). Another example is the image of the World from the viewpoint of Ronald Reagan (composed by journalists): Fig. 4.

The book is constructed in such a way that the reader is offered to pass from simple methods of compilation of anamorphosises to more complicated ones, as if restoring by that the historical chronology of development of this field of science. In the beginning of the book the method of compilation of rather simple linear anamorphosises is shown. For area anamorphosises at first manual methods of compilation are described. Then there are methods of mechanical analogy, of electric simulation and the photographic method. This part ends with a variety of numerical methods of compilation of anamorphosises. The last chapter gives some examples of use of anamorphosises in Earth sciences.

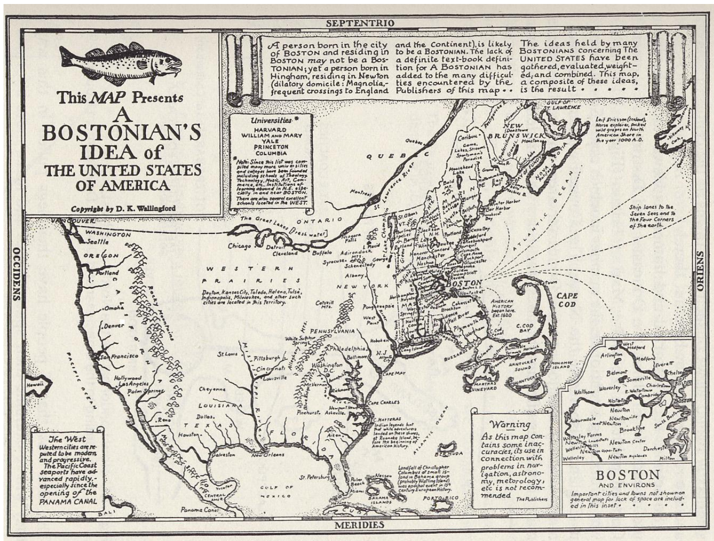


Fig. 3: USA from the viewpoint of Boston residents

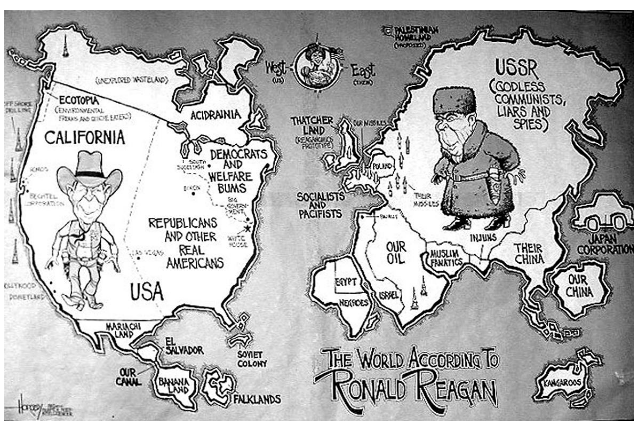


Fig. 4: World from the viewpoint of Ronald Reagan

Chapter I. Compilation of linear anamorphosises.

I.1. Transformation of the length scale.

Among anamorphated images one can distinguish linear, area and volumetric ones ones. Moreover all of them can be animated. Linear anamorphosises often look like graph images. Changes of lengths of edges of them permit to change the distances between the regarded units (vertices) depending on values of characteristics taken as the base of the anamorphosis. Examples of linear anamorphosises are: image of system of Moscow subway lines reflecting the accessibility of stations measured in time expenditures (Salishchev, 1982), image of distances from shops to a fixed point of a city (for example - from its center), measured in time expenditures again (Murdych, 1983), image of export-import connections of the former USSR with countries of the Europe (Vardomskiy, Tikunov, 1982) and so on. In all these examples certain spatial relations are preserved, in contrast to statistical graphics consisting of columns of different heights.

First linear anamorphosises appear in the middle of the past century. Most of them were compiled on the base of the time. As the illustration of the simplest linear anamorphosis, which reflects only the order of location of stations and which is compiled manually, the scheme of system of Moscow subway lines can serve (Fig. 5). In other known to us examples geometrical distances from one initial point of the map to all other points are replaced by conditional distances measured in time or material expenditures (Bunge, 1966; Salishchev,

Fig. 5: The scheme of Moscow city subway system as an example of a linear anamorphosis.

1973; Monkhouse, Wilkinson, 1971; Murdych, 1969, 1971; Ewing, 1974, Muller, 1978 etc.) Sometimes such transformations are produced from maps of isochrones. On the anamorphotic image they are organized in a system of concentric circles (see Fig. 6).

As an example, in his well-known book "Theoretical Geography" W.Bunge showed the map of "real" distances (measured in time units) from the city center of Seattle (after E.Kant).

There are known anamorphosises produced on the base of logarithmic, hyperbolic, parabolic or another change of geometrical distances from a fixed point (center) (Trunin, Serbenyuk, 1968; Hagerstrand, 1957; Kadmon, 1975; Murdych, 1983). Fig. 7 shows time expenditures for flight by a plane and for journey by a train from Moscow to capitals of former socialist countries, to capitals of former republics of the USSR and to centers of largest economic regions. Directions (azimuths) from the center to other points correspond to azimuths on the map of the world in the Mercator projection, directions are shrunk or stretched depending on the accessibility in time. Fig. 8 shows the location of glass and porcelain shops in Prague in the logarithmic scale (Murdych, 1983). Fig. 9 shows the outline of Sweden (A) and its transformation to the azimuth logarithmic projection centred at the Asby city (B), and also the migration field, created in the representation of migrants from the Esbu Asby county about other regions of the country (C). At present there have been worked out a variety of methods of transformation of distances (Tikunov, Yudin, 1987; Murdych, 1983; Muller, 1979, 1982, 1983; Muller, Honsaker, 1980 and so on.

As an illustration let us use the example of compilation of linear anamorphosis for the town-building analysis (Tikunov, Yudin, 1987). Complexity of orientation inside the complicating space of cities distinguishes the time as one of the main factors of coming to decisions. The increase of movements between residential places and working places in combination with the modernization of means of transport changed our representation and perception of the space. The settlement is guided not by the geometric distances from residential places to the places of permanent application of labour, but by the time, which is needed for the overcoming of this distances by available means of transport. To the question: "How far from your working place do you

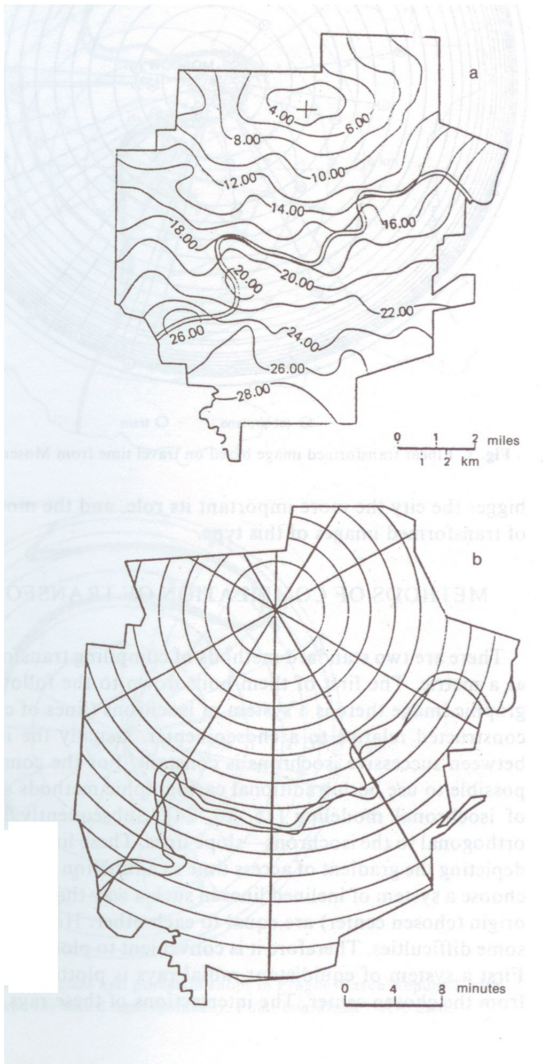


Fig. 6: Isarithmic map of travel time (in minutes) by car from a fixed point (marked by the cross) in Edmonton, Alberta, Canada (a). Anamorphated image in which the positions of city boundaries and the course of the Saskatchewan River are transformed under the (travel time) metric of Fig. a (b). Lines of equal time of travel (in minutes) by a car from a fixed point (marked by the cross) in Edmonton: A — The initial map, in the south-west part of which one can see two isolated islands formed by inversions of the slope; B — the image in the equalarea (isometric) polar projection, where the deformations of the city boundaries and of the Saskatchewan valley are caused by the transformation of the space as a function on the time (after Muller, 1983).

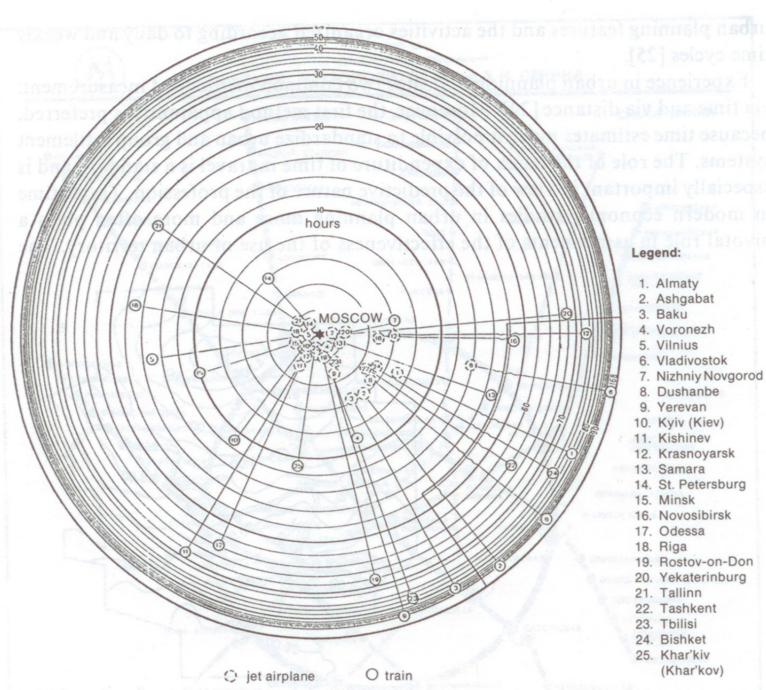


Fig. 7: Linear transformed image based on travel time from Moscow to nearby and remote cities. Accessibility from Moscow to capitals of former Union republics and other big cities of the USSR: 1 — Alma–Ata, 2 — Ashkhabad, 3 — Baku, 4 — Voronezh, 5 — Vilnius, 6 — Vladivostok, 7 — Gorki, 8 — Dyushambe, 9 — Yerevan, 10 — Kiev, 11 — Kishinev, 12 — Krasnoyarsk, 13 — Kuibyshev, 14 — Leningrad, 15 — Minsk, 16 — Novosibirsk, 17 — Odessa, 18 — Riga, 19 — Rostov–on–Don, 20 — Sverdlovsk, 21 — Tallinn, 22 — Tashkent, 23 — Tbilisi, 24 — Frunze, 25 — Kharkov. a — by plane, b — by train (after Trunin, Serbenyuk, 1968).

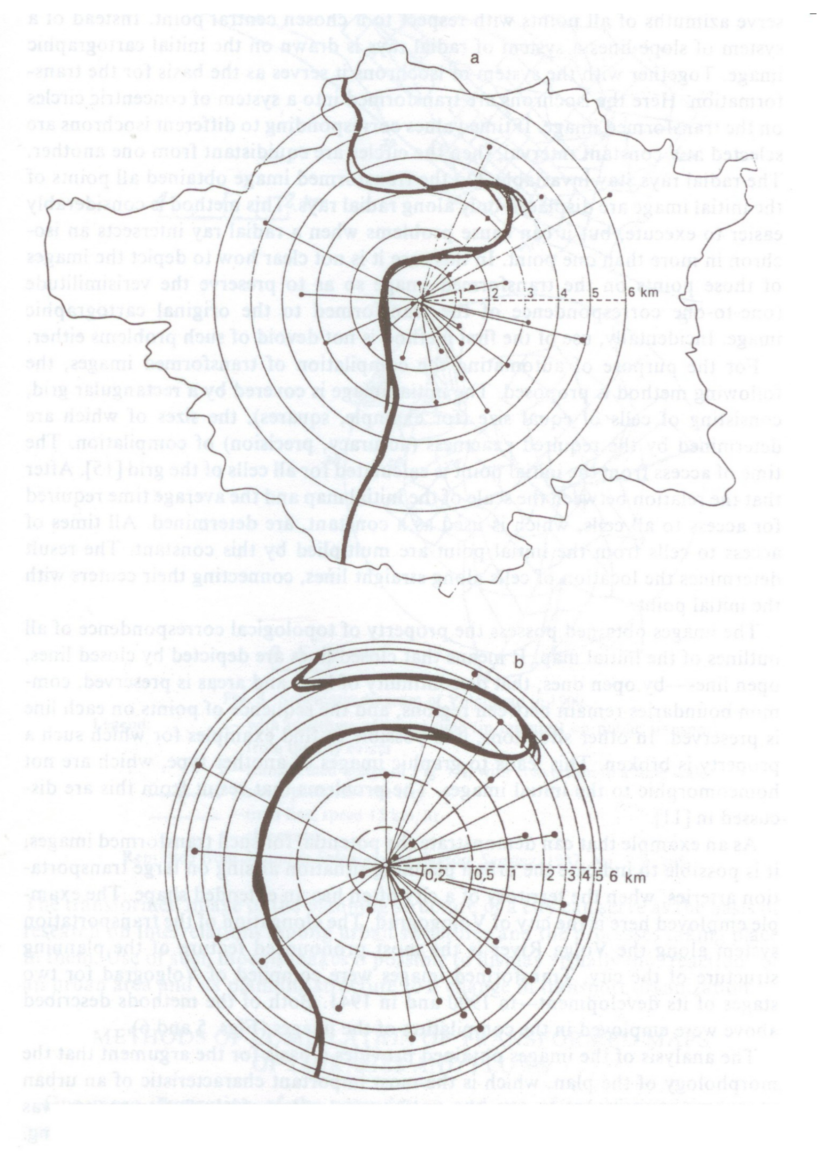


Fig. 8: Distribution of glass and porcelain shops in Prague, Czech Republic: A — on a conventional map; B — on an anamorphosis compiled in the logarithmic scale of the distance (after Z. Murdych, 1983).

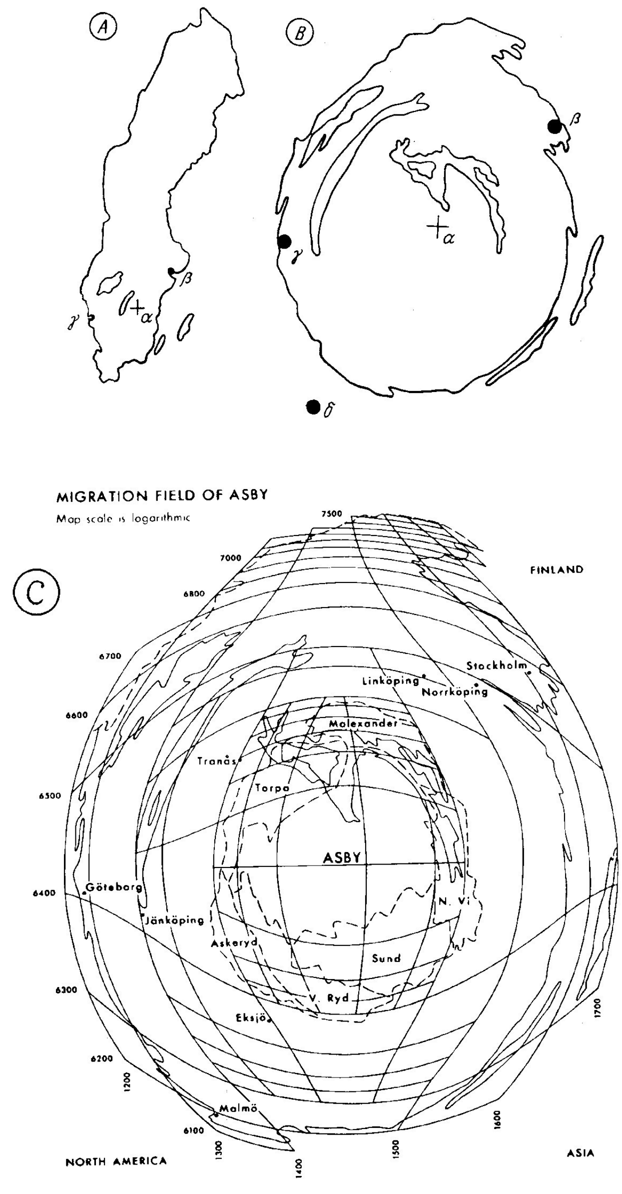


Fig. 9: Transformation of the image of Sweden (A) into the azimuthal logarithmic projection (B), centred at the city of Asby (α) and the migration field, which gives a representation about view points of migrants from the Asby county on other country regions (C).

live?" - a townsman most often will answer that he lives, for example, 20 minutes journey, but not in 7 kilometers and so on.

The space of settlement has a much more complicated character, and residents of cities percept the space which is considerably different from that one which is shown on traditional maps or plans of cities. Nowadays a number of researchers (Ewing, Wolfe, 1977, 1983; Muller, 1983) try to solve the problem of graphic representation of transformations (shrinking of some parts of the city space and stretching of others), which are brought in by the distribution of transport, by artificial and natural obstacles (Ewing, Wolfe, 1977, 1983; Muller, 1983).

There can be distinguished as minimum three aspects of purposeful regard for the time by the analysis and planning of town-building objects. The first of them arises by the analysis of a city on the level of its whole evolution, i.e. by the analysis of the historical formation on different levels of development of the society. Then the time factor is regarding on the level of decades by the analysis of interactions between town-building characteristics of objects and the activity of population in the process of formation and development of the city. At last there is the regard for the time on the level of functioning of the city by the analysis of relations between town-building structures and the processes of activity, which are organized in frames determined by the balance of the time, based on daily and weekly cycles (Yargina, 1982).

The experience accumulated in town-building has led to two methods of measuring: by the time and by the distance (Yakshin et al., 1979). Moreover the first method has evident preference, because time estimates give possibility for standardization of urban and group settlement systems. The role of the index of time expenditures for movement in the general system of estimates is respectively constant. It is especially important in view of the forecasting character of the town-building planning. Today in economic studies in town-building the index of time expenditures more and more often plays the role of one of important criteria of estimation of effectiveness of use of the city territory. It testifies for growth of its importance. The bigger is the city the more important this factor is for it. The presence of transport makes estimations with the help of the index of (geometrical) distance inadequate.

The index of time expenditures for transport movement is not only a valuable characteristic of one of local municipal systems. It can be used for complex estimation of the whole territorial-planning organization of a city. As a matter of fact just the transport forms the real spatial-temporal structure of a city. The transport infra-structure characterizes not only the level of development of the transport net, but also specific features and potentialities of the city plan, as the base for realization of most important functions of the activity of population and subsequent development of the city. So the use of the "scale" of time seems to be well-founded enough in town-building investigations, what determines the base for advisability of compilation of corresponding anamorphated images.

There are two main methods of compilation of anamorphosises, which use time as the metrics of the images. The first of them reduces to the following. On the initial cartographic image there is constructed the system of isochrons (lines of constant time accessibility) with respect to the chosen center. Usually the values of time corresponding to different isochrons are chosen with the constant step (difference). For the construction of isochrons it is possible to use both traditional cartographic methods and mathematical methods of isochrons modelling. (Poponin, 1972; Yakshin et al., 1979; Petrov, Tikunov, 1984). Then there are plotted orthogonal to isochrons trajectories - lines of incline. These lines can be also described as lines of gradient of the time of access as a function on the plane. It seems natural to choose the system of lines of incline in such a way that the angles between them in the origin (chosen center) are equal to each other. However such an approach meets some difficulties. Therefore it is convenient to draw them in the following way. At first there have to be drawn the system of equidistant radial rays on the initial image, coming from the chosen center. The intersections of these rays with a fixed isochron (usually with the most distant one) are taken as initial points for construction of the lines of incline.

If the isochrons of the initial map are transformed into circles on the anamorphosis, the lines of incline are transformed into the system of radial rays, coming from the chosen center with the zero time of accessibility. So on the anamorphosis there turns out a system of polar coordinates, which fixes positions of any elements of the initial image in the unique way.

Another (somewhat more simple) method is used when it is necessary to preserve azimuths of all points with respect to the chosen central point. Instead of the system of lines of incline the system of radial rays is drawn on the initial cartographic image. Together with the system of isochrons it serves as the base for the anamorphation. In this case isochrons are transformed into a system of concentric circles on the anamorphosis. If the values of time corresponding to different isochrons were chosen with the constant step, then these circles are equidistant from each other. The radial rays stay invariable. On the obtained anamorphosis all points of the initial image are displaced only along radial rays. This method is considerably easier for realization, but it can cause difficult problems in the case if a radial ray intersects an isochron in more than one point (in three, five ...). In this case it is not clear how to determine the images of these points on the anamorphosis in order not to preserve the uniqueness (one-to-one) of the transformed cartographic image. Incidentally the first method also can meet just the same difficulties.

For the purpose of automation of compilation of anamorphosises it is possible to offer the following method. The initial image is covered by a regular net, consisting of equal cells (for example, of squares), the sizes of which is determined by the required exactness (accuracy, precision) of compilation. The time of access from the initial point is calculated for all cells of the net (Petrov, Tikunov; 1984). After that there is determined the relation between the scale of the initial map and the average time of access of cells, which is used as a constant. All times of access of cells from the initial point are multiplied by this constant. The result determines the displacement of cells along straight lines, connecting their centers with the initial point.

The obtained images possess the property of topological correspondence of all outlines of the initial map. It means that closed lines are depicted by closed lines, open lines - by open ones, there is preserved discontinuity of lines and areas, presence or absence of common boundaries between regions, without any inversion there is preserved the succession of points on each line. In other situations it is possible to find examples for which such a property is broken. It leads to graphic images of another type, which are not homeomorphic to initial images. Difficulties arisen by that are discussed in the paper (Muller, 1983).

As an example, which can show the possibilities of such anamorphated images, it is possible to indicate the town-building situation arising on big transport ways, when the territory of a city often has a stretched shape. The example - the city of Volgograd. With respect to the transport system just the stretchness along the Volga-river is the most characteristic feature of the planning structure of the city. There were compiled anamorphosises of Volgograd for two stages of its forming and development - in 1900 and in 1941. For compilation of anamorphosises both described methods were used (Fig. 10 and 11).

The analysis of the obtained images permits to assert that the morphology of the plan, which is the most important characteristic of a town-building object and constitutes the architectural side of its solution proper, transforms considerably during the process of its exploiting and functioning. The anamorphated image of the planning structure of a city can serve as the base of investigations, directed at the expansion of representations about interrelations of town-building structures and processes, taking place in them. Use of such models gives possibility to display visually the "reaction" of a town-building object and of its planning structure on a change of transport organization, i.e. to model the dynamics of its functioning in traditional for an architect-town-builder terms.

Investigations, connected with the search and exposure of potentialities of anamorphated images, can serve as the initial stage of working out the description of a city, which is able to reflect adequately main features of structure, functioning and development of real town-building objects. Moreover the work, connected with non-traditional types of images (as anamorphosises in town-building), is a preparation for their introduction into the town-building analysis as an additional tool. Such images will permit to raise trustworthiness of results of investigations and will contribute to the improvement of methods of town-building analysis.

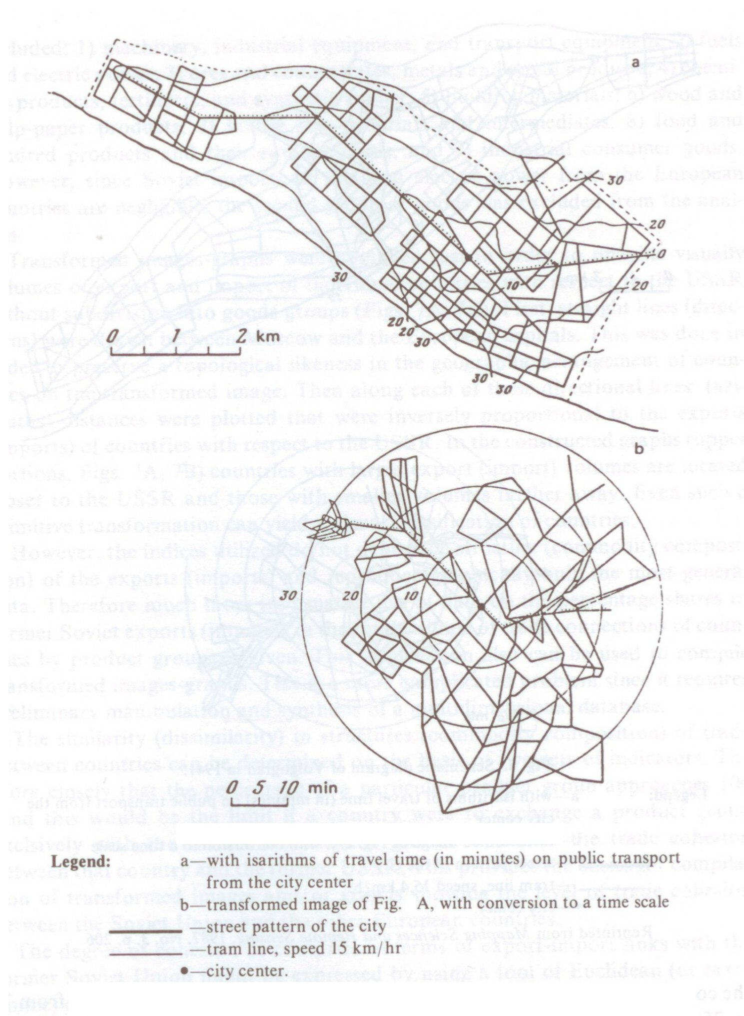


Fig. 10: Schematic plan of Volgograd, 1990; a — with isolines of access time (in min.) by public transport from the city centre; b — the anamorphosis of the image *a* with the transition to the "time scale". ____ — the city street net; ...... — tram lines with the speed 15,4 km/h; • — the city centre.

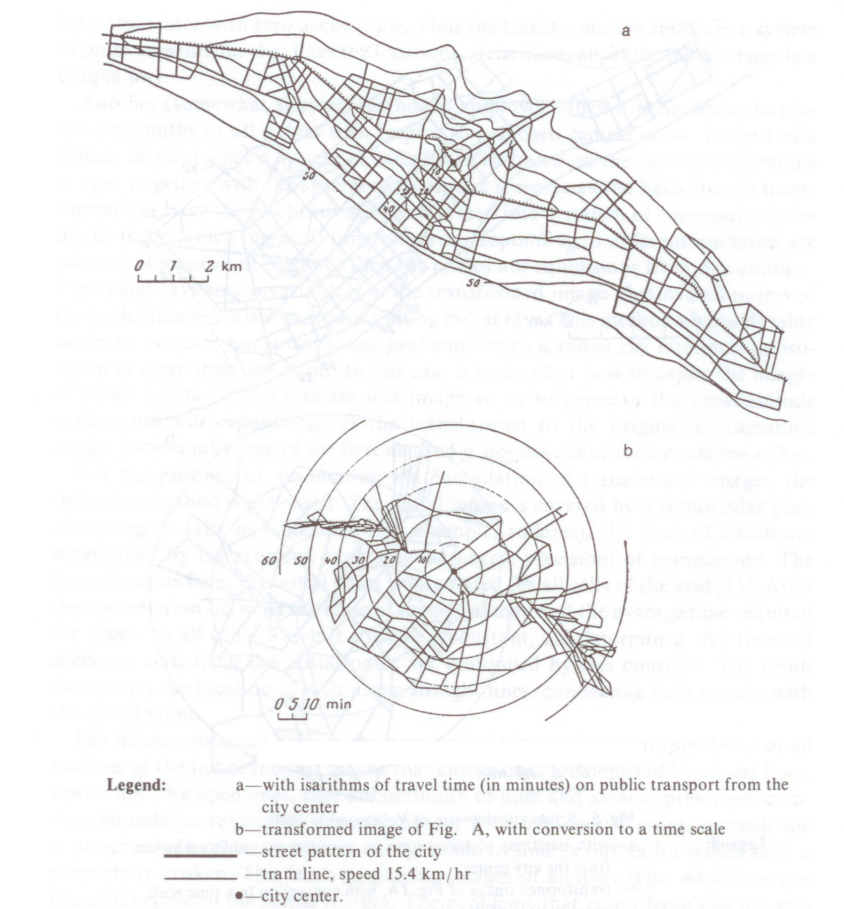


Fig. 11: Schematic plan of Volgograd, 1941; a — with isolines of access time (in min.) by public transport from the city centre; b — the anamorphosis of the image *a* with the transition to the "time scale". ____ — the city street net; ...... — tram lines with the speed 15,4 km/h; • — the city centre.

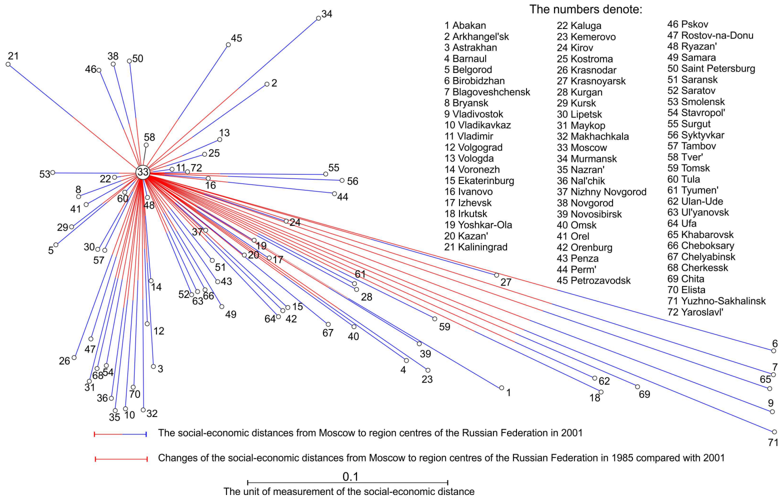


Fig. 12: Linear anamorphosis of change of the social-economic distance from Moscow to the centers of Russian Federation for the railroad passenger communications in 1985 and 2001.

As an example of characterization of time changes let us give linear anamorphosises (Fig. 12, 13) which show changes of mutual transport remoteness of regions of Russia (D.V.Malinovsky, V.S.Tikunov, A.I.Treivish, 2002). On them, there are drawn symbolic straight (azimuth) lines from the center of Moscow connecting it with each center of a subject of Russian Federation (and also with Surgut since Khanty-Mansijsk (the administrative senter of the Khanty- -Mansi Autonomous Region) does not have a rail-road station). Then the value of the "price distances" from Moscow (Fig. 12) and to it (Fig. 13) in 1985 and 2001 are put on these lines with the chosen. As a result one gets linear anmorphosises depicting not only the "price" remoteness, but also its change.

I.2. Compilation of linear anamorphosises on the base of synthetic characteristic

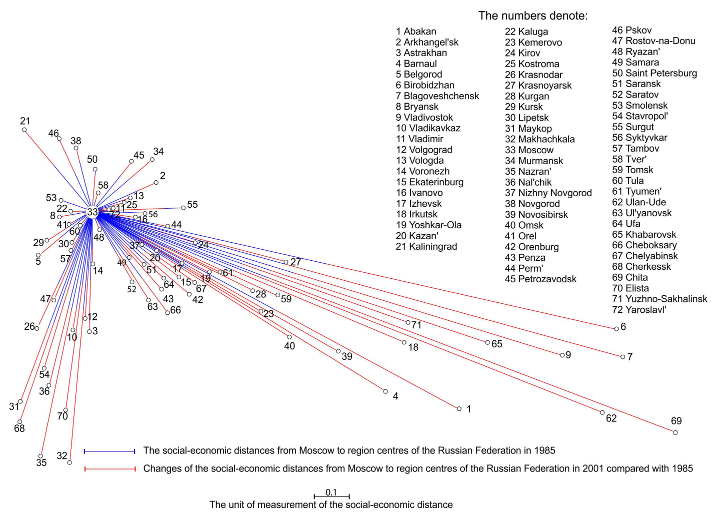


Fig. 13: Linear anamorphosis of change of the social-economic distance from the centers of Russian Federation to Moscow for the railroad passenger communications in 1985 and 2001.

To illustrate compilation of linear anamorphosises let us use the example of construction of anamorphosises-graphs characterizing export-import connections of the former USSR with European countries (Vardomski, Tikunov, 1982). Trade with the countries of Europe accounts for 70 to 75 percent of the trade of the USSR, and Europe includes the Soviet Union's most important trade partners. Trade with these countries has been distinguished by a diversity of products and by a long history. At the same time, Soviet trade with particular European countries has varied widely both in the volume and in the commodity composition of exports and imports. (The statistical data used in this paper are taken from Vneshnyaya torgovlya SSSR v 1978 g., the Soviet foreign trade yearbook for 1978, and from Stiltisticheskiy yezhegodnik stran-chlenov Soveta Ekonomicheskoy Vzaimopomoshchi, 1978, the Comecon foreign trade yearbook for 1978.) Both the quantitative level of foreign trade and its qualitative composition reflect to a some extent the degree of trade cohesion between countries. This cohesion, in turn, reflects the level of complementarity of economic structures of the USSR and its trade partners.

It goes without saying that the potential for such complementarity is particularly pronounced between the USSR and the Eastern European countries, and is being realized in the course of the international socialist division of labour and socialist economic integration. But even among the socialist trade partners of the USSR, there are differences in the intensity and diversity of trade, reflecting the sectoral structure and the size of the economy, the specific character of national interest and existing economic policies. A subject that might be at the focus of the geographical study of foreign trade is a typology of trade cohesion between the USSR and the European countries, which can be investigated by the use multidimensional mathematical models.

For this purpose, the author calculated the share of Soviet exports in the imports of the European partner countries and the share of exports from the European countries in the imports of the USSR as of 1978 for the following product groups: (1) machinery, industrial equipment and transport equipment; (1) fuels and electric power; (3) ores and concentrates, metals and metal products; (4) chemical products, fertilisers and synthetic rubber; (5) building materials; (6) wood and pulp-paper products; (7) textile raw materials and intermediates; (8) food and kindred products and their raw materials; (9) consumer goods. Soviet imports of fuel and electric power from the European countries were negligible and were therefore ignored.

For more clear impression, Soviet exports to and imports from each of the European countries (without a breakdown by product groups) are shown in the form of a value-based distance graph (Fig. 14). This was done, first, by drawing vector lines between Moscow and the European capitals with a view to preserving a topological likeness in the geographical arrangement of countries. Along each of these directional lines, the authors then plotted distances inversely proportional to the exports (imports) so that countries with larger export (import) volumes would be located closer to the USSR and those with smaller volumes farther away. Even such a primitive graph can yield a visual classification of countries.

It is natural that the used indices do not reflect the export (import) structure completely and show graphically only yje most general data. A more useful data base is provided by the percentage shares of Soviet

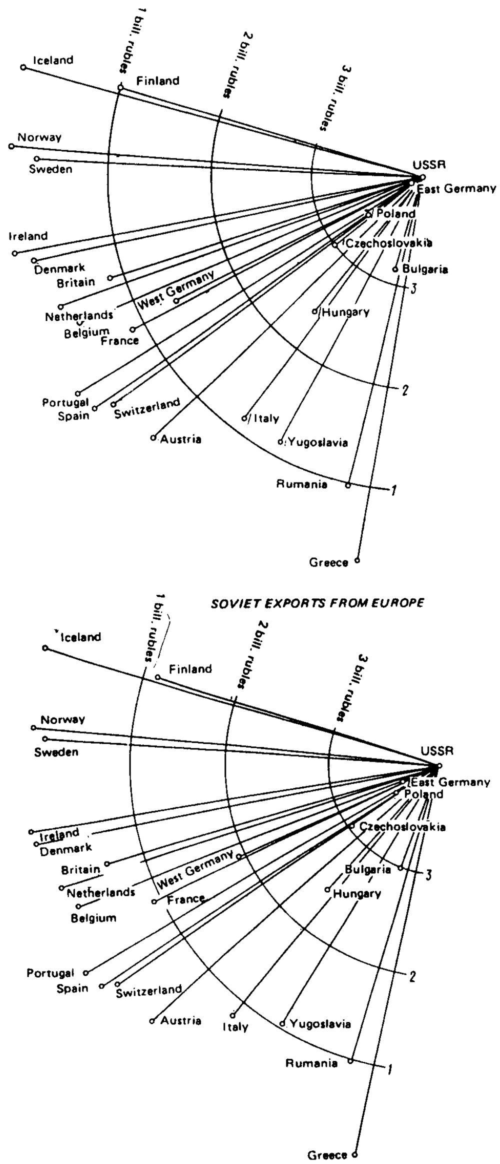


Fig. 14: An anamorphosis based on the absolute volumes of foreign trade connections of the USSR with European countries.

exports (imports) in the trade of the partner countries by product groups. This information can also be used to construct value-based graphs. This is a more complex problem since it requires preliminary synthesis of a multidimensional data base. On the other hand, the relative complexity of the computations is balanced by the fact that such a graph would show structural differences in both exports and imports.

The similarity (dissimilarity) of structures in trade relations between countries can be determined on the basis of two sets of indicators. While these indicators would be expressed in absolute values for all countries, the Soviet share would, of course, have to be expressed in percent. The closer the percentage of a particular product group approaches 100 (and this would be the limit if a country were to exchange a product group exclusively with the USSR), the greater would be the trade cohesion between that country and the USSR, and this makes it possible to use the indicators of structural similarity to construct graphs showing the level of trade cohesion between the Soviet Union and the other European countries.

The degree of similarity of countries in terms of export-import links with the Soviet Union might then be expressed by using a tool of multidimensional mathematical analysis known as Euclidean distances, or taxonomic distances. These distances are derived by processing a matrix of initial data by the well known algorithm of principal component analysis (see Harman, 1972; Zhukovskaya, Muchnik, 1976; Zhukov, Serbenyuk, Tikunov, 1980 etc), what makes it possible to present a system of indicators in orthogonal form and to "compress" it by excluding from further calculations the last components accounting for a negligible percentage of the variance. In our own experiment, we used the first two principal components in processing the Soviet export matrix and the first three components for the Soviet import matrix, accounting for more than 90 percent of variation of the initial sets of data. Attempts to use, instead of the Euclidean distances, other indices of similarity of objects in a multidimensional space do not give more reliably interpretable results.

Then, by using the formula for taxonomic distances in the system of measurements of the component space, we calculated the distances from the USSR of all the points symbolizing the European countries. As in Fig. 14, the resulting vector-columns of taxonomic distances for both exports and imports can be represented in the form of graphs, what was made in Fig. 15 and 16. It should be noted that the relative length of taxonomic differences varies because there were used 9 product groups for exports and 8 product groups for imports. In contrast to Fig. 14, these graphs do not display sharp contrasts among countries because the export (import) structures are unlikely to be highly specialized. Although some specialization occurs, it is less sharply differentiated than the difference in total trade volumes.

The created anamorphosises use a single measure to express the types of cohesion between the Soviet Union and the European countries in exports (imports) based on the relative shares of deliveries in the various commodity groups. It may therefore be useful, for subsequent geographical analysis of the graphs, to compile maps of the types of countries in terms of export-import linkages with the USSR and to analyse the graphs together with the traditional typological maps. This can be done by simply using matrices showing the export (import) shares of European countries relative to the USSR without any data relating to the Soviet Union.

The data in the matrix were also processed by the algorithm of principal component analysis, with 90 percent of the variance accounted for by the first three components for export data and the first four for import data. The resulting orthogonalized data were used to calculate symmetrical matrices of taxonomic distances, which were then used to break down the multivariate algorithm for a typological differentiation of the set of countries into heterogeneous taxon: groups (3). The model we used was based on the criterion of determining maximum differences between internally homogeneous groups of countries in terms of the structure of exports (imports). On the basis of both the export data and the import data, the algorithm was worked out for nine alternative classifications of countries by varying the number of toons from 10 to 2. For each alternative classification, we calculated the special coefficients of homogeneity reflecting the quality of the grouping, which then allowed us to select the final, statistically optimal alternatives of classification shown on the maps:

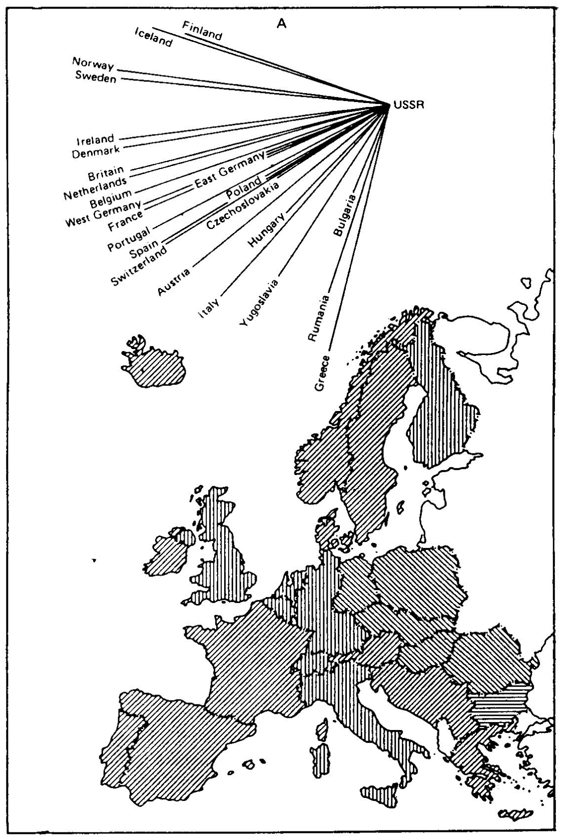


Fig. 15: Anamorphosises and suplementary maps of export types of the USSR to European countries (the sense of the groups 1–4 is explaned in the text).

Fig. 15 and Fig. 16.

The first group of countries on the map of Soviet exports to Europe includes only Bulgaria, whose structure of imports from the Soviet Union differs somewhat from that of the countries in Group 2. Bulgaria is characterized by the very high degree (up to 80 percent) to which its import requirements are met by purchases from the Soviet Union in the entire range of product groups except for foodstuffs and kindred products and their raw materials. In general, the USSR is the principal supplier not only of industrial materials and fuels, but also of manufactured goods to Group I and Group 2, both of which are made up of the member countries of Comecon. This is the basic difference between these first two groups and Groups 3 and 4.

Group 3 includes the most important capitalist trade partners of the USSR: West Germany, Finland, Italy, Britain, Switzerland, Belgium and the Netherlands — all of which were major consumers of Soviet fuels and raw materials. The Soviet share in the imports of energy goods by these countries ranges from 5 to 10 percent. In other product groups, the Soviet share in imports is usually less than one percent. The exception is Yugoslavia, which actually holded a position intermediate between Groups 2 and 3 (which is evident in the graph), but on the basis of the calculations falls in Group 3.

Group 4 was made up of the other European countries, whose trade with the Soviet Union was smaller and not so diversified. The exceptions here are France, Austria and Sweden, which also seem to hold an intermediate position, between Groups 3 and 4.

In a similar way, the countries were classified on the basis of their exports to the Soviet Union. Here again four country groups can be distinguished in terms of the type of commercial relations with the USSR. Group I consists solely of Bulgaria; Group 2 contains the other socialist countries of Eastern Europe plus Finland; Group 3 is made of West Germany, Italy, France, Britain, Austria, Sweden, Switzerland, Belgium and the Netherlands, and Group 4 includes all the other European countries.

Summing up this little experiment, it might be noted that the classification of countries in terms of the closeness of commercial linkages made it possible to generalize a large body of highly diversified data.

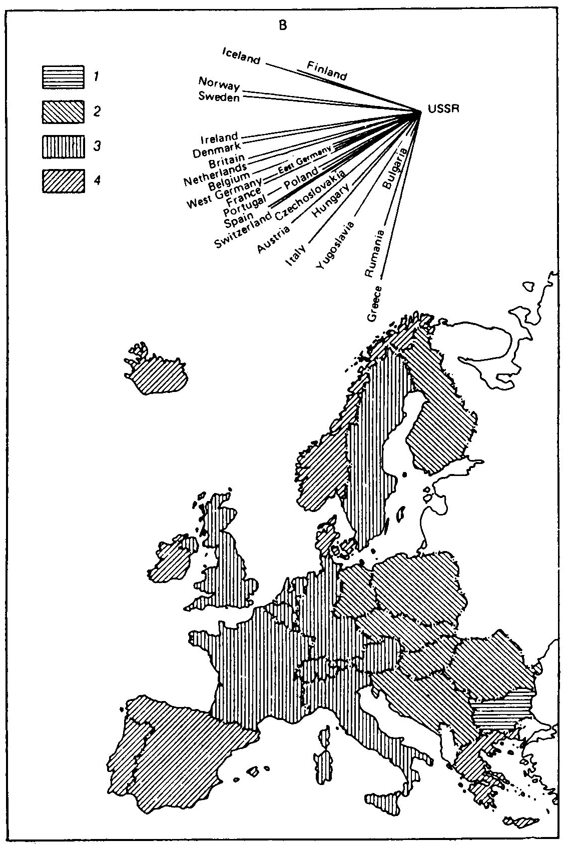


Fig. 16: Anamorphosises and suplementary maps of import types of the USSR to European countries (the sense of the groups 1–4 is explaned in the text).

...

Fig. 17: Functional remoteness of cities of the Central Economic Region from the stadard — Moscow.

...

Fig. 18: Functional remoteness of cities of the Moscow oblast from the stadard — Moscow.

It should be borne in mind, however, that the inadequacy of data in some cases and the use of data for only one year may affect some of the details of the graphs. But on the whole, the results seem to conform to what is known about the export-import linkages between the USSR and the European countries.

In the second part of the experiment one made a classification of cities of the Moscow oblast (Gornostaeva, Tikunov, 1988, a, b) on the basis of the employment data in 8 sectoral groups: 1) industry; 2) construction; 3) transport and communications; 4) agriculture and timber industry; 5) trade, public nutrition, state procurements, and material and technical supplies; 6) public health, physical education, social security, education, science, and culture; 7) housing, municipal, and consumer services; 8) administrative organs.

In this case all cities were ranked by the rate of their difference from Moscow. It appeared to be convenient to represent the result in the form of a graph, lengths of whose edges are proportional to distances in the space of indices which characterize the difference from Moscow. Directions of the edges approximately coincide with those from Moscow to corresponding cities. On the ends of the edges there are circles shading inside which shows types of the cities (Fig. 17, 18).

For sure the obtained results needed a detailed interpretation. First of all there were analyzed transitions of cities from a group to a group in due course. The second problem was determination of changes of the functional structure of cities of a certain type with respect to the structure of Moscow: if they become more similar or more different.

The most close by their structure to Moscow was the group of regional polyfunctional oblast centres of the Central Economic Region (CER). They had a more "homogeneous" structure of functions: the ratio of the industry here is lowered in comparison with the average for the region at the expense of a higher ratio of non-production sectors. At the same time the employment rate in the industry in this group of cities was much higher than that in Moscow. Regional polyfunctional centres were sufficiently mobil: in course of the time there appeared more "transitions" than in any other group. E.g., by 1980 Kaluga had passed from the group of centres with developed construction functions to the group with a significant localisation of the industry. To the same group there had passed Ryazan' from the group with the average employment ratio in the industry. The group of cities "average" in the sense of the employment ratio in industry also had been enriched. Ivanovo had passed to this group from the group of superindustrial regional centres, Kalinin — from the group of developing centres with a increased level of costruction. By 1981 the situation had changed again: Orel had passed from the group of "developing" centres to the group of superindustrial ones, Smolensk had moved from the group of "developing" centres to the group of "average" ones, Vladimir had made the shift in the opposite direction. The cities of this class most functionally close to Moscow were (in the order of decrease of the closeness): Vladimir, Kalinin, Ivanovo, Smolensk, Kostroma. Moreover, in course of the time the first three of them gradually approached the structure of Moscow, but the last two became functionally more different from the standard (see Table 1).

Centres with primary developement of industry functions usually are far from Moscow by their functional structure. The only city from this group similar to Moscow is Naro–Fominsk, the structure of which approximates one of the standard. In course of the time cities of this class do not change their structure essentially, moving on th whole inside this class. For instance, by 1984 there became disbanded the subclass of industry centres with essential development of agriculture and timber industry: Noginsk has moved to the group of superindustrial centres, Egor'evsk and Naro–Fominsk have moved to the subgroup of industrial centres with developed construction functions, Balashikha has moved to the group of suburban industrial centres with essential development of service functions. Kimry and Shuya have moved to the same subgroup, however they have higher employment rate in industry and lower one in service sectors.

In 1970th-80th three industrial–transport centers: Dmitrov, Shchekino, Kineshma — moved to the group of industrial centres, i.e., their rank in the functional hierarchy has increased. In 1980–84 Zagorsk and Zheleznodorozhnyj moved from the class of cities with exceptionally high development of service sectors to the group of industrial centres with essential development of service functions.

By 1980 the class of industrial–construction centres was reinforced by Novomoskovsk (earlier it was an industrial–transport centre, by 1984 it was reinforced by Dmitrov and Shchekino (these cities have moved from the group of industrial centres with developed construction functions, i.e., their construction functions have increased). By structure all cities of this class are not close to Moscow.

The class of industrial–transport centres was not reinforced by new cities; essentially it included cities which were on the initial stage of the functional development, except Orekhovo–Zuyevo which was close enough to Moscow by its functional structure. This was explaned by the fact that it was a polyfunctional centre and its attribution to the industry-transport group was connected with a relatively high portion of the transport functions.

The class of cities with exceptionally high portion of the employment in service sectors was the most close to Moscow and includes the capital itself. By 1980 this class was reinforced only by one city — Zheleznodorozhnyy, which moved here from the class of industrial-transport centres, but by 1984 the city had left this group. The cities of this class mostly close to Moscow were Reutov, Odintsovo, Khimki.

The classification of cities of the Moscow oblast also had brought to light the following tendencies. By 1984 several cities had moved from the groups of industrial–construction centres and centres with developed agriculture and timber industry to the group of superindustrial centres. A number of centres became industrial-construction ones (Mozhaysk, Shatura, Elektrougli, Vysokovsk, Shcherbinka). Moreover the functional structure of some cities from this class was sufficiently similar to the functional structure of Moscow (see table 1).

The group of industrial–transport centres was reinforced by an industrial–construction centre Ivanteevka, Taldom had moved from the group of transport–industrial centres to the group of cities — district centres with developed transport and construction.

The group of industrial centres with essential developement of service sectors had changed considerabely: by 1984 there entered Aprelevka, Istra, Skhodnya and other cities. The functional structure of Istra was very close to that of the standard.

It should be noted that cities moved from a group to a group not only because of a change of their own functional structure, but also because of transformation of functional structures of all cities of the class under consideration. For instance, there was a general tendency to lowering of the employment rate in the industry and grouth of the employment rate in service sectors.

Some cities with the number of population over 50 thousand related to different classification groups in classification of the cities of CER and of the Moscow oblast. For example, in the classification of cities of CER in 1980 Naro–Fominsk was included into the group of industrial centres with developed agriculture and timber industry, but into the group of industrial centres with essential development of service functions in the classification of cities of the Moscow oblast. This was connected with the fact that the employment rate in agriculture and timber industry in cities of the Moscow oblast on the average was higher than in cities of CER with the number of population over 50 thousands, and therefore inside the Moscow oblast the localisation of this function in Naro–Fominsk was not essential, but it was appreciable in CER. There were also other non-coincidences as well, connected with different average functional structures of cities of CER and of the Moscow oblast.

Table 1 permits to see the level of closeness of functional structures of cities of the Moscow oblast and of CER with the functional structure of the standard and also to estimate the changes in closeness of this structures in due course. It is clear that the accepted method of estimation of the closeness of functional singularities selects into the group of cities mostly similar to Moscow polyfunctional centres which had "uniform" structure of functions, what was expressed, in particu-

Dynamics of functional distances for two periods 1970/1980, 1980/1984.	"Distance" (functional) of cities from Moscow in 1984 (similarity of structures).			
	very similar	similar	non-similar	very non-similar
1	2	3	4	5
Stable rapprochement with the standard structure	Vladimir, Ivanovo, Zheleznodorozhnyy, Reutov Ivanteevka, Orekhovo–Zuyevo, Naro–Fominsk,	Tula, Voskresensk, Domodedovo, Egor'evsk, Klin, Ramenskoye, Ozery, Balashikha, Kolomna, Shchekino, Shchelkovo, Rzhev, Zaraysk, Aleksandrov, Efremov, Noginsk, Lukhobitsy, Skhodnya, Taldom	Vysokovsk, Khot'kovo, Drezna, Krasnoarmeysk, Ozherel'e.	Solntsevo, Obninsk,
Extinguishing rapprochement which can lead to the divergence of structures	Odintsovo, Khimki, Volokolamsk, Zvenigorod	Aprelevka, Shcherbinka, Shuya, Lyubertsy, Bronnitsy, Solnechnogorsk	Kurovskoe, Stupino, Pushchino, Klintsy, Zhukovskiy, Vreya	Likino–Dulevo, Losino–Petrovskiy, Gus'–Khrustal'ny.

Table 1:

1	2	3	4	5
Rapprochement of structures started after their divergence	Kalinin, Istra, Elektrostal', Chekhov	Yaroslavl', Dmitrov, Kashira, Novomoskovsk, Kimry, Andropov, Safonovo	Lytkarino, Fryazino, Dubna, Yakhroma, Pavlovski Posad Uzlovaya, Vyaz'ma	Dzerzhinski, Krasnozavodsk
Divergence of structures started after their rapprochement	Vidnoe	Pushkino, Ruza, Vyshni Volochek	Lobnya, Roshal', Vichuga	Elektrougli, Aleksandrov, Kovrov
Fading divergence, which can be changed by rapprochement of structures	Smolensk, Shatura	Kaluga, Orel, Ryazan', Krasnogorsk, Bryansk, Podol'sk Elektrogorsk, Klimovsk, Kineshma	Troitsk	Murom
Stable divergence with the structure of the standard	Kostroma, Mozhaysk	Mytischi, Zagorsk, Dedovsk, Srpukhov	Dolgoprudnyy, Yaroslavl	Kaliningrad

Table 1: Continuation.

lar, in a lower employment rate in the industry and in a higher one in service sectors. Cities with such similar structures, and in particular those which in course of the time became more close to the standard, could serve as kernels of development of competitive functions. First of all this took place for oblast centres of CER. Cities which had functional structures close to the Moscow one and moreover were situated in a vicinity of Moscow were in fact a continuation of it and in long-range outlook they could be considered as a part of Moscow.

A large group of cities functionally similar to Moscow in the territorial sense occupied an intermediate position between oblast centres which pretended to play the role of counterbalances and cities — satelites of Moscow. They were, for example, Mozhaysk, Volokolamsk, Zvenigorod, Shatura etc. Such cities — district centres of the Moscow oblast — copied the polyfunctionality of Moscow on their level. Independence of their functional structure was caused by a higher than for a number of other cities isolatedness, remoteness from the capital. Growth of their attractiveness could be connected with a certain deformation of their "uniform" structure, for istance, by the growth of the employment rate in the industry or in the construction sector.

Those not similar to Moscow were primarily unifunctional cities: industry ones, scientific etc. However this did not prevent these cities from being satelites of Moscow (Dolgoprudnyy, Zhukovskiy) and even from becoming a part of it (Solntsevo). Here the principle of supplementation of structures acted: part of their functions were suppressed by functions of closely situated Moscow. It was in the favour of the fact that one could not ignor the role of small and medium unifunctional centres which were situated far from Moscow and which accepted one urban function of Moscow: sientific (Dubna, Obninsk, Pushchino, Troitsk) or advanced industrial one; in future, as a possibility, also the educational (construction of high education centres) etc. However the full-bodied execution of the functions of Moscow by these cities was possible only in the presence of their reliable connection (transport, information) with the main city, if the quality of the city environment is close to the Moscow one, what would permit to make closer not only living standards, but also the level of urbanization of the style of living of residents of the centre and of the outlying areas.

The analysis of the dynamics of the functional structures of cities of CER and of the Moscow oblast is a part of complex studies of cities of this region. The obtained results can be used for the choice of the strategy of development of the territorial structure of the Moscow region.

Chapter II. Analogue methods of compilation of area anamorphosises

II.1. Manual methods of compilation of anamorphosises

Among anamorphosises the most widely distributed are area ones, which make uniform the density of an index (for example, the density of population, the density of the territorial distribution of incomes, the density of the territorial distribution of consumption of a product and so on). It means that in this case areas of images of territorial units become proportional to corresponding values of the index which is in the base of the anamorphosis. It is demanded that to a possible extend anamorphotic images preserve arrangement of territorial units with respect to one another, their shapes etc. Among compiled anamorphosises mostly often are found equidemical images, for which areas are proportional to numbers of residents of corresponding territories. More rarely can be found anamorphosises in bases of which their lie gross revenue of population, gross yield of grain, gross national product and so on.

The history of compilation of area anamorphosises numbers several decades. First known to us attempt of transformation of a cartographic image goes back to the beginning of the century. In Germany there appeared an original cartographic production, which belonged to German cartographer H.Wiechel (Kartogramm..., 1903). He prepared an anamorphosis to illustrate results of Reichstag elections. On this image ratios of sums of areas, distinguished by definite colors, were equal to ratios of numbers of votes given for candidates.

M.Eckert described the essence of such a map transformation in the following way: "A relief model of the population density, the height of which in each point corresponds to the density of population, is being "rolled" until it becomes a sheet of a constant thickness, equal to the average density of population. Parts of the model with bigger height, which correspond to bigger density of population, move apart horizontally. Therefore neighbouring parts, corresponding to less populated regions, first of all are also moving and secondly under the action of the horizontal pressure they will be driven to the height, corresponding to the average density of population. As the result of flattenings and displacements there is obtained a cartogram with the density of population, which is constant in all its parts." Then M.Eckert makes a pessimistic conclusion: "The method has not found followers. The enthusiasm exited by the appearance of Wiechel's maps, is already over" (citing according to Vasilevskiy, 1970). He turned out to be wrong. There occurred to be quite a number of followers of Wiechel from different countries: Australia, Israel, China, New Zealand, Poland, Russia, USA, Czechoslovakia, Switzerland and so on (Harris, McDowell, 1955; Tobler, 1963, 1979; Murdych, 1969, 1971; Elsasser, 1970; Cerny, 1972; Dent,1975; Angel, Hyman, 1976; Olson, 1976; Sen, 1976; Sliwa, 1978; Griffin, 1980; Kadmon, 1982; Kelly, Neville, 1985; Vakhrameeva, Bugaevskiy, 1985; Tikunov, 1986; Bugaevskiy, 1987; Hua Yi-xin, 1989). It is not difficult to continue this list list of names with the help of bibliography in the end of the book.

Nevertheless, starting from the "population cartogram" of Wiechel and till the eighties, the majority of known anamorphosises were compiled basically by hand. Thus, as long ago as in the thirties E.Raisz (Raisz, 1934, 1936, 1948) offered the method by which the anamorphated image is constructed as a system (net) of connected rectangles which are in one-to-one correspondence with units of administrative division (for example, of countries). Moreover, the arrangement of these rectangles corresponds to (in some sense, approximates) the real arrangement of the units. As on any area anamorphosis the areas of rectangles are directly proportional to corresponding values of the regarded index (for example of the number of population). The chosen system (arrangement) of rectangles is assumed to be preserved by representing different indices. Such images were constructed for representation of steel production, of textile and power sectors of industry, of population and of agricultural production. The author called the obtained images "statistical cartograms".

Later A.Getis (Getis, 1963) offered the following method. The initial image is covered by a net of connected quadrangles (a priori does not corresponding to any "natural" division of the territory). For cells of this net there are determined total values of the index lying in the base of the anamorphosis. The total area of the obtained configuration is preserved, and the compilation is carried out by a transformation of the quadrangles into rectangles in the following way. The lengths of the sides of quadrangles along one of the coordinate directions (say along the vertical one) are not changed, the lengths along the other direction (horizontal one) are changed in such the degree, that there areas become directly proportional to the values of the regarded index.

An analogous method was used by J.Pravda (Pravda, 1983a, 1983b) for representing the distribution of population of Slovakia. The initial cartographic image was not covered by a net of quadrangles. Instead of it there were constructed rectangles with areas directly proportional to the given values. The arrangement of the rectangles on the plane was determined by the configuration of the initial image, which was in some sense approximated by these rectangles (Fig. 19).

It is natural, that the precision of anamorphosises, compiled with the mentioned methods, is low enough. Moreover, it is Obvious, that in such a situation there are as many possible results as compilers. Sometimes the obtained image loses its topological similarity with the original. Points of intersections of boundaries turn into lines, adjoined territories lose the property of neighbourship. Territorial units are substituted by rectangles or other simple geometrical figures. Such methods are rather statistical, results of them are nearer to diagrams than to cartographic images.

Sometimes territorial units are intentionally substituted by rectan-

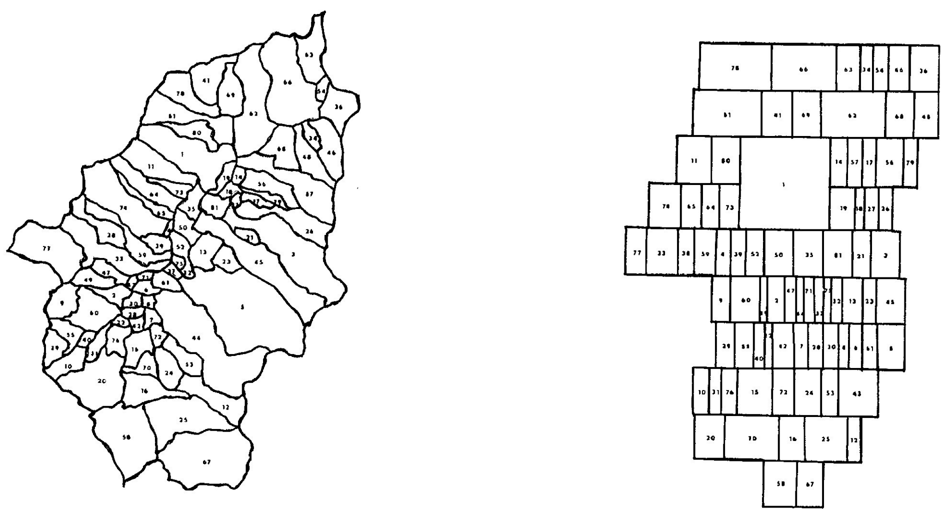


Fig. 19: The example of transformation of the Martin district (after J.Pravda, 1093).

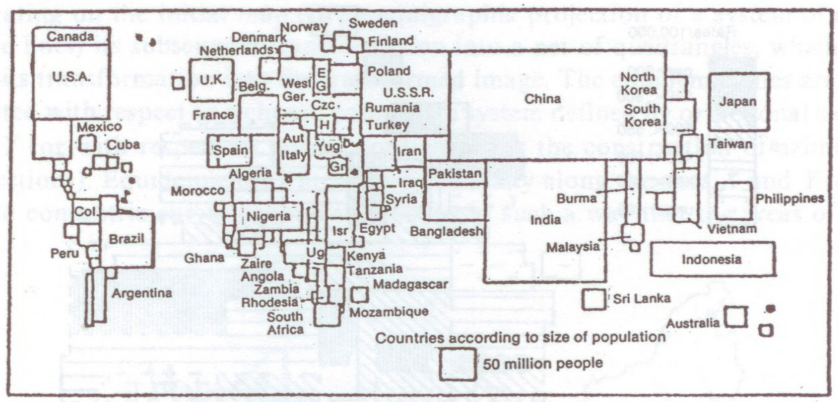


Fig. 20: Image of world countries based on the number of population (according the atlas The Library...,1980).

gles or other simple geometrical figures (Bochkareva, 1981; Suvorov, 1984a,1984b; Cuenin, 1972; Murdych, 1983 et al.). A.K.Suvorov (1986) supports the proposition, that in fact territories are not represented on anamorphosises, and therefore it should not be tried to preserve configurations of initial images. Thus, from his point of view, maps of the world in the forms shown on Fig. 20 and 21 are more preferable in comparison with, for instance, starting from the very first examples (Fig. 2, 3, 22) and so on (Fig. 23, 24), the anamorphosis also manually compiled (Fig. 17). One can give a lot of such examples, however it is interesting that even on relatively early examples of (Fig. 25, 26). anamorphosises of both types one can meet additional substantial characteristics suitable for geographical analysis. Moreover on the last picture there is outlined a depart from the purely rectangular forms to preserving configurations (though in a very rough form). This effect is stronger on more precise images. As an example we show the anamorphosis compiled in the Geography Institute of the University Vrije (Brussels, Belgium): Fig. 27.

For these purposes A.K.Suvorov offered a method, a result of applying of which is shown on Fig. 28. In our opinion, solving one and

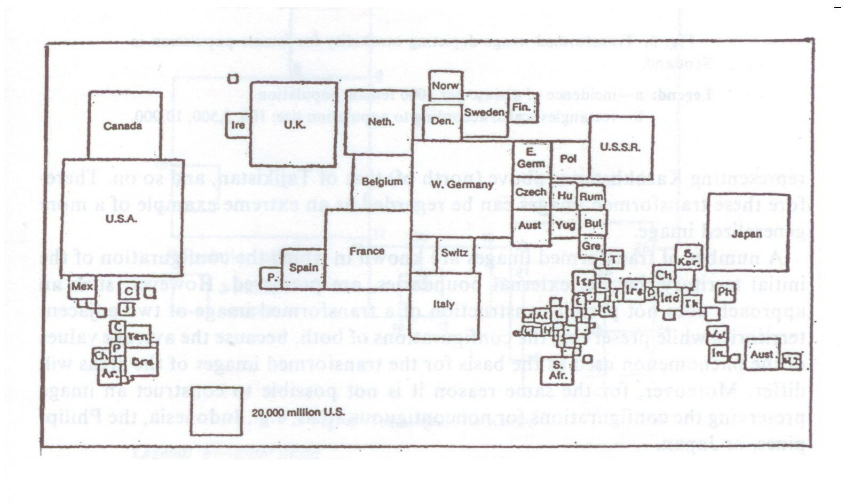


Fig. 21: The image of the export and import volumes (in millions USD) (based on the atlas Philips'...,1976).

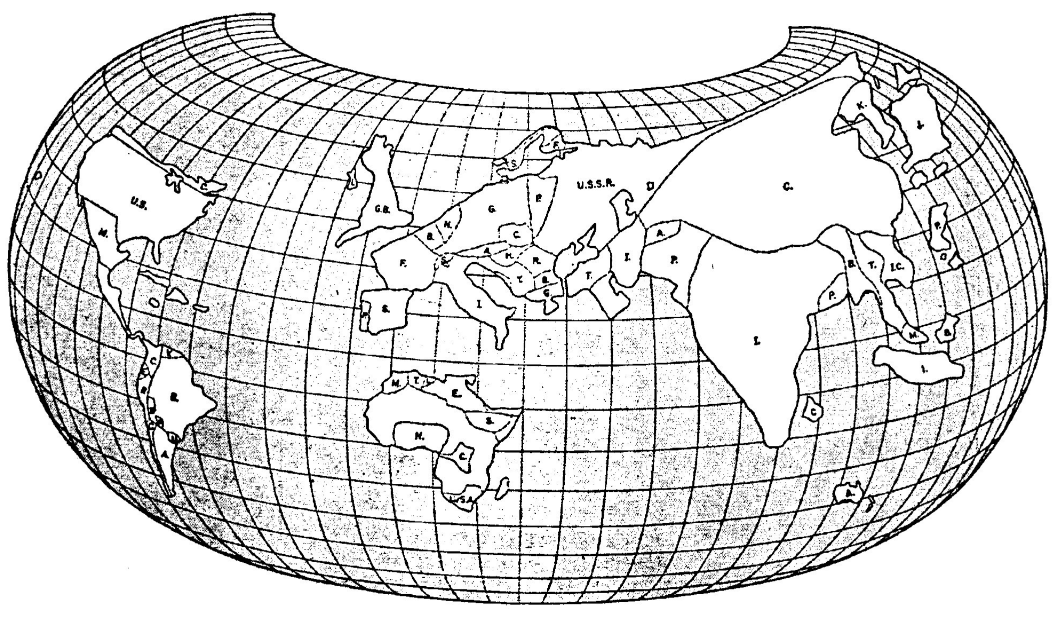


Fig. 22: Image of the World by Voitinski brothers.

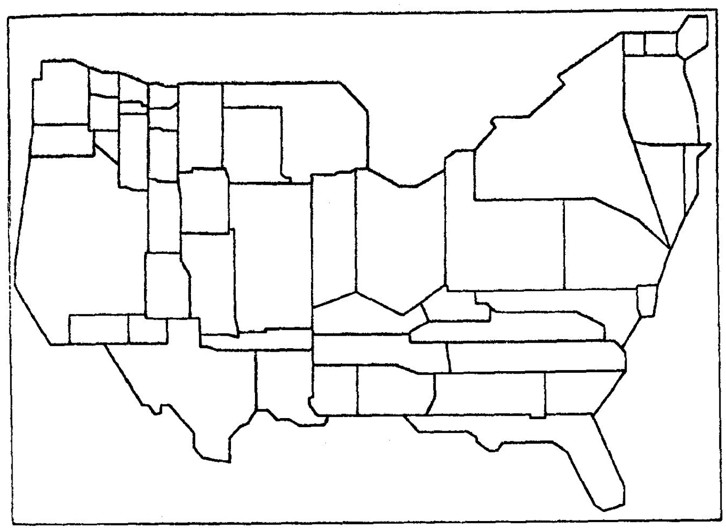


Fig. 23: Image of the USA (by P.Muehrcke).

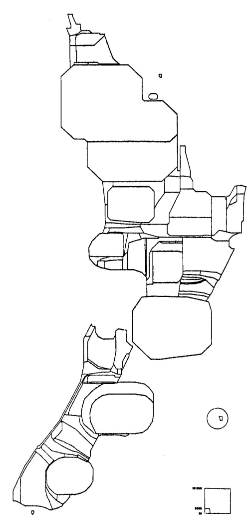


Fig. 24: The image of the New Zealand (after Kelly, Neville, 1985).

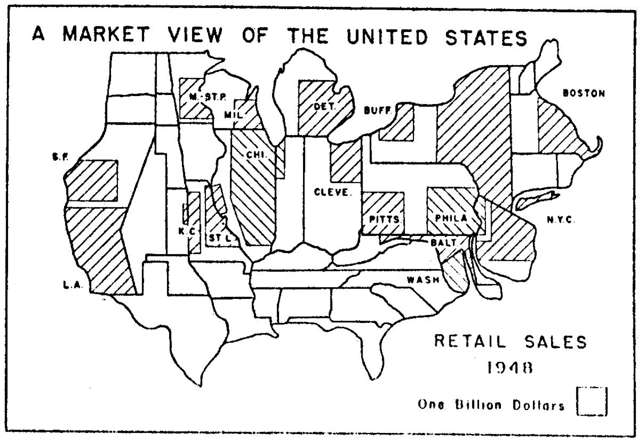


Fig. 25: Distinguishing of cities on the anamorphosis of the USA (after C.D.Harrison)

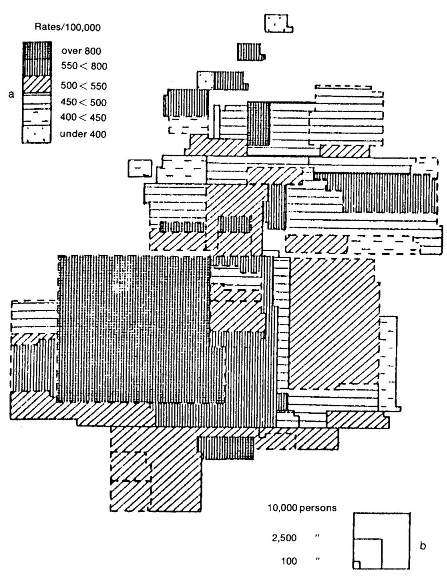


Fig. 26: Representation of the risk of diseases in Scotland; a — number of diseases per 100 000 people, b — sizes of rectangles are proportional to the numbers of population: 100, 2 500, 10 000 (after Forster, 1966).

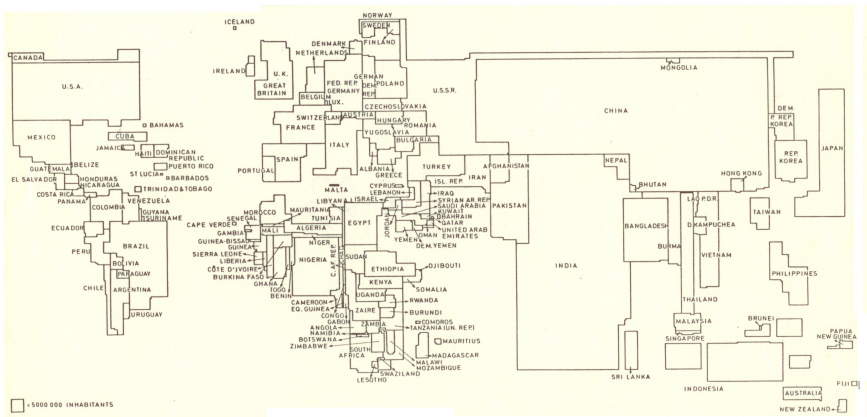


Fig. 27: Countries of the World from the Medical–geographical atlas compiled in the Geography Institute of the Vrije University (Brussels, Belgium).

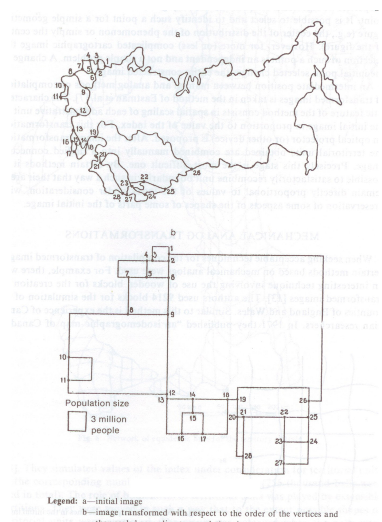


Fig. 28: Topological nets: A — the initial one, B — the one topologically transformated with respect to the order, and then by the number of population (after Suvorov, 1986).

the same problem - to represent areas of countries directly proportional to number of population - the last image is more preferable owing to the possibility of recognition of countries by a reader. In more details it is described in the review of the book of A.K.Suvorov (Tikunov, 1988), where, in particular, is criticized the constant emphasizing by A.K.Suvorov of the impossibility to compile anamorphosis on the base of the traditional cartographic image and to compare them. From our point of view, part of the content of traditional maps and spatial relations on them, important for geography, must be in the base of anamorphosises and therefore anamorphotic images can (and even have to) be compared with traditional cartographic images.

Assuming the possibility of showing on traditional maps of any phenomena or processes, not connected directly with the topographic metric, for instance of economic linkages of factories, A.K.Suvorov criticizes attempts of some authors (J.Pravda, P.M.Polyan, A.I.Treyvish) to represent elements of content of general geographical maps in the metrics of (connected with) social-economic characteristics (Suvorov, 1986, p.p.81, 113). Just searching for methods to preserve configurations of initial images was the aim of a number of researchers (Lacko, 1967; Vasilevskiy, 1970; Hunter, Meade, 1971; Dent, 1972; Jackson, 1974; Borden, 1975, 1976; Sliwa, 1978; Eastman, Nelson, Shields, 1981; Griffin, 1983; Selvin et al., 1984; Dougenik, Chrisman, Niemeyer, 1985; Tobler, 1986; Brassel et al, 1991). There are a number of papers specially devoted to this problem (Dent, 1972; Griffin, 1980, 1983 etc.).

If real spatial relations are not reflected in the arrangement of the geometric figures symbolizing territorial units (see Fig. 20, 21, 26, 29 and others), then these images must be regarded as cartoids. In the mentioned papers geometric figures are situated with taking into account of real arrangement of corresponding territorial units with respect to each other (Fig. 28) [The square of Kazakhstan is above that of Tadgikistan (Suvorov, 1984) and so on] and therefore these anamorphosises can be regarded as the extreme case of mostly generalized image.

In some cases such images become a sort of rebuses, what can be seen on the Fig. 29. The reader is encouraged to identify countries

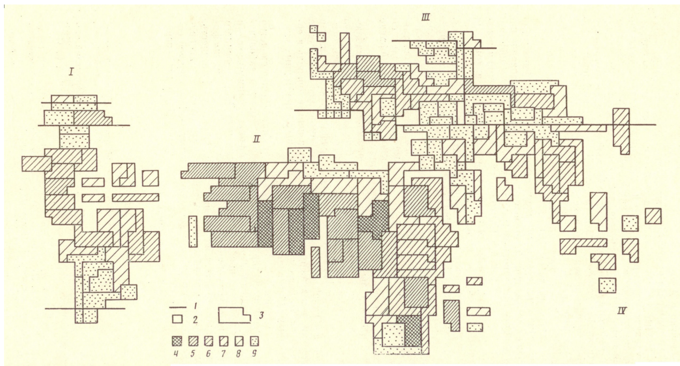


Fig. 29: Topological image of the geographical distribution of characterristic nature-endemic diseases of the population. I — America, II — Africa, III — Eurasia, IV — Australia. 1 — boundaries of natural zones; 2 — one characterristic nature-endemic disease coded in a symbolic form: as the area of the square; 3 — topological image of a country or an individual territory with the number of characterristic nature-endemic diseases equal to 7. Number of everywhere spread diseases in the structure of all characterristic nature-endemic diseases of population: 4 — 5 or more; 5 — 4; 6 — 3; 7 — 2; 8 — 1; 9 — there are no everywhere spread diseases (after Raykh, Maksimova, 1988).

on it on his own. Let us note that on this figure, taken from the paper (Raykh, Maksimova, 1988), there is shown the distribution of nature-endemic diseases of population by two indices the total number of diseases and the number of those, spread everywhere. Areas correspond to total number of deaseses, the intensity of the shading reflects the number of diseases spread everywhere. On the other hand even such complicated by their configuration and contrasting by changing of areas territories, shown on Fig. 30, 31, and 32 can be recognized on anamorphosises "preserving shapes" of territorial units. Just "recognition" of shapes permits geographers to perceive anamorphosises as something suitable for an analysis, not as a complete abstraction.

There are known a number of anamorphosises, by the compilation of which the configuration of the initial territory, i.e. the outline of it is preserved. However such an approach does not permit to con-

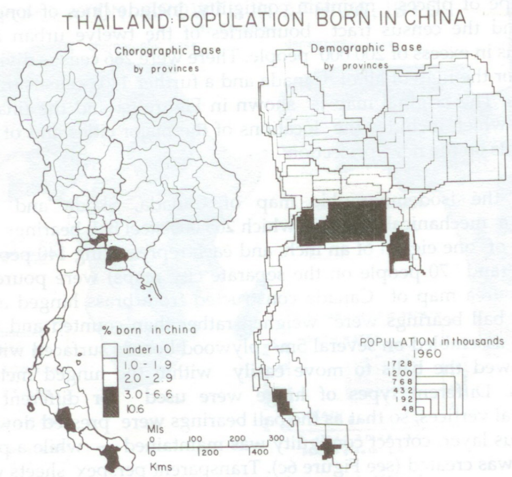


Fig. 30: The initial (A) and the anamorphated (B) images of the ratio of residents of Thailand born in China; a — the ratio of the population (per cent) born in China; b — the number of population in thousands (after Hunter, Meade, 1971).

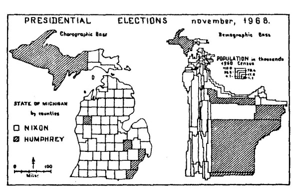


Fig. 31: The initial (A) and the anamorphated (B) images of the election results in Michigan, a — the number of population in thousands (after Hunter, Meade, 1971).

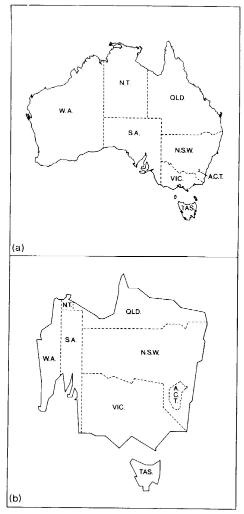


Fig. 32: States and territories of Australia based on census 1976 data (after Griffin, 1983).

struct an anamorphosis of two adjacent territories with preservation of configurations of both of them, because the average values of the phenomenon in the base af the anamorphosis for these two territories will be different. Moreover, due to the same reason in this case it is not possible to construct an anamorphosis with preservation of configurations for non-connected territories, e.g. for Indonesia, the Philippines or Japan.

For the purpose of objectiveness of compilation of anamorphosises L.I.Vasilevskiy (1970) offered a simple method. The essence of it consists in drawing on the initial map in the equigraphic projection of a system of equidemic lines, the subsequent transformation of which into a net of quadrangles permits to transform the the initial map into the anamorphated image. The equidemic lines are constructed with respect to a chosen coordinate system defined by orthogonal axes X and Y (or with respect to polar coordinates for the construction of azimuthal projections). Equidemic lines are drawn manually along the axes X and Y (or as closed concentric curves around the center) in such a way (with the condition) that the areas of cells bounded

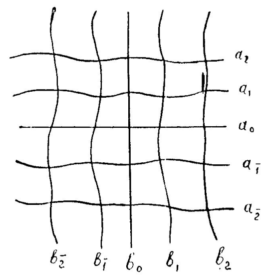


Fig. 33: Equidemic net on the initial image in the rectangular system of coordinates (after Vasilevskiy, 1970).

by them would be inversely proportional to values of the index in the basis of the anamorphosis.

As the whole, the net which is drawn on the initial map looks like the one shown on the Fig. 33. See also the net on the initial map of Italy (Fig. 34). In the plane of the anamorphosis this net corresponds to the usual coordinate net (Fig. 35). For azimuthal anamorphosis the net shown on the Fig. 36 is constructed. In this case in the plane of the anamorphosis it corresponds to the regular net consisting of radial axes (just the same radial axes as in the net on the initial map) and of equidistant concentric circles, which correspond to irregular closed curves of the initial net.

Constructions of Vasilevskiy, as all formalized constructions, possess the properties of strictness and uniqueness, but only for the fixed choice of the initial point. It is possible to choose and to fix such a point for a simple geometric figure (it is possible to take the center of distribution of the phenomenon or simply the center of the figure). However for more or less complicated cartographic image the choice of such a point is an independent and not simple problem. The change of the initial point changes the whole constructed image.

An intermediate position between manual and analogic methods of compilation of anamorphosises occupies the method of Eastman et al. (Eastman, Nelson, Shields, 1981). The characteristic feature of the method consists in spatial scaling of each administrative unit of the

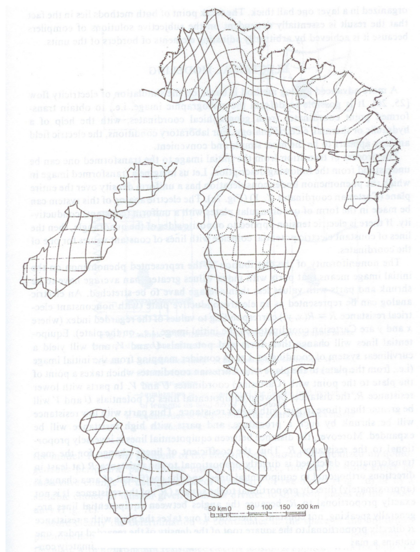


Fig. 34: Equidemic net for the territory of Italy (after Vasilevskiy, 1970).

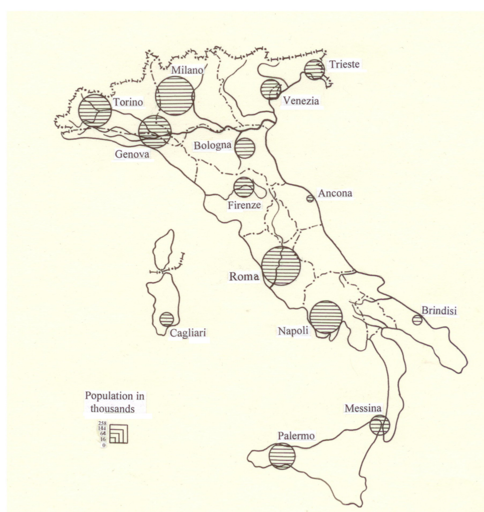


Fig. 35: Anamorphated image of Italy. 1 — the contemporal state border, 2 — the border in 1931, 3 — borders of historical regions (after Vasilevskiy, 1970).

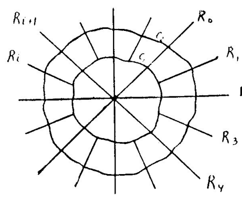


Fig. 36: Equidemic net for azymuth constructions (after Vasilevskiy, 1970).

initial image in proportion to the value of the index. For this transformation it is offered to use the optical projector (or other devices). After the transformation the obtained territorial units are combined manually into a united connected image. Just this stage is the most difficult one. It is offered to overcome the difficulties by acquisition of some skills of such a combination. By definite habits it is possible to achieve a satisfactory recombination of contours, the areas of which stay directly proportional to values of the representing index, with preservation of some features of shapes of both pats of the initial image and the image in whole. The final stage consists in general correction of the image.

II.2. Methods of mechanical analogy in compilation of anamorphosises

When looking for acceptable techniques of compilation of anamorphosises some methods based on mechanical analogies were used. For example there was an interesting technique with the use of wooden blocks for creation of anamorphated images (Hunter, Young, 1968). The authors used 9214 blocks for imitation of 62 counties of England and Wales.

Similar to this method is the experience of canadian researchers. In 1971 they published "an isodemographic map of Canada" (Skoda, Robertson, 1972). They simulated values of the index under consideration for territorial units by the corresponding numbers of 1/8" diameter steel balls (256 thousands of balls were used in total). The role of boundaries of territorial units was played by extensible partitions.

As territorial units there were used Census districts of 1966. Moreover the boundaries of these regions were preserved only if the population exceeded 100 000. Twelve urban territories were maped separately with maximal possible accuracy in details of boundaries of Census districts. The alternation process of boundaries of Census districs for the Alberta province can be seen on Fig. 37 and 38. Boundaries of districs were straighten with simultaneous fixation of nodal points.

The size of the used balles determined the generalization degree. Each ball corresponded to 70 inhabitants of urbanized territories. The Census districts were filled in by the metal balls according to their population. Moreover, the extensible partitions were moved in such a way that, to a possible extend, shapes of territorial units were preserved. The initial packing of the balls is shown on Fig. 39. The result was obtained when the the ball arrangement became single-layered. The initial and the obtained images of Canada are shown on Fig. 40. The authors claimed that the lack of financial resources did not permit to make the image more perfect.

A weak point of both methods consists in the fact that the result is essentially dependent on subjective solutions of compilers because it is achieved by adjusting elements of borders.

2.3. The method of electrical simulation

A more advanced in comparison with the mechanical analogies is the method, based on use of electric simulation (Raspolozhenskiy, Sventek, Tikunov, 1972, 1974). It is possible to deform the cartographic image, i.e. to obtain anamorphated co-ordinates of points from their geographical co-ordinates, with a help of hydraulic, thermal or electric analogy. However for reproducing in laboratory conditions electric field appears to be the most simple and convenient.

The process of transition from the initial image to the anamorphated one can be understood from the following description. Let us

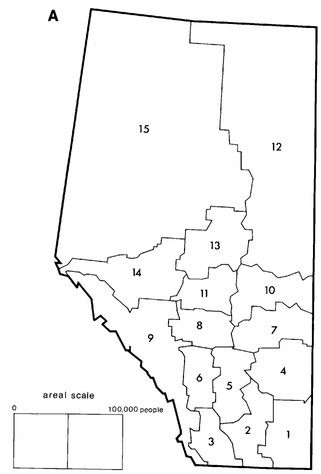


Fig. 37: The initial cartographic image of the Alberta province.

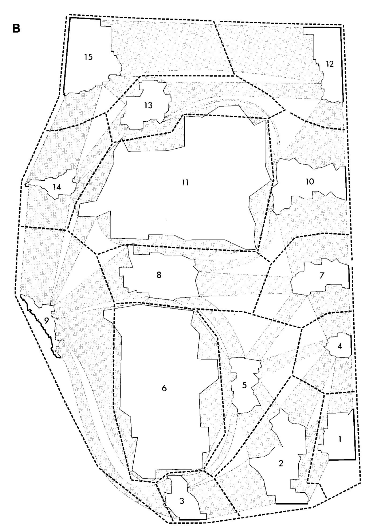


Fig. 38: The image of the Alberta province anamorphated by the number of population.

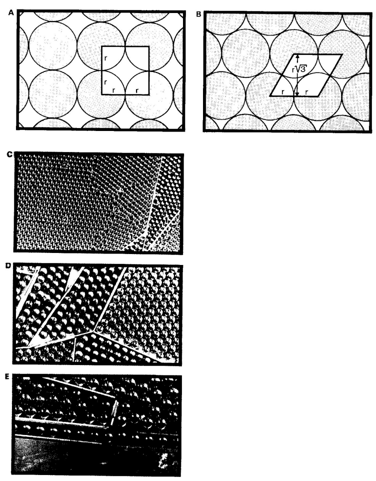
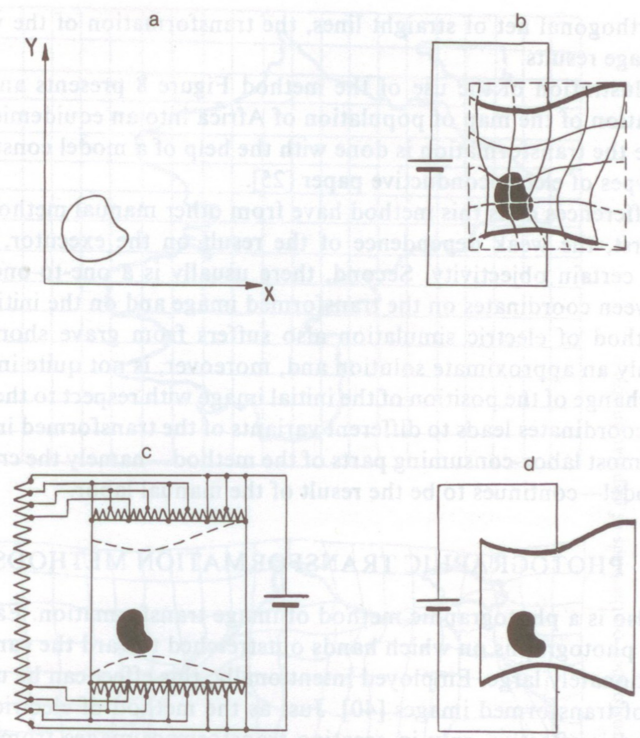


Fig. 39: Packing of balls: A — the arrangement for which (by a statement of L.Skoda and J.C.Robertson) there is the maximal possible area for each ball; B — the arrangement for which there is the minimal possible area for each ball; C–E — photoes of pieces of the model by the construction of the anamorphosis.

Fig. 40: The initial and the anamorphated images of Canada (after Skoda, Robertson, 1972).

imagine the obtained image (anamorphosis) in which the phenomenon under consideration has a uniform density over the whole plane with Cartesian coordinates X and Y in it (Fig. 41-a). An electric analogue of this system can be made in the form of quadrangular plate with a uniform (constant) conductivity. If there is electric tension applied to opposite sides of the quadrangle, then the lines of constant electric potential coincide with lines of constant values for one of the coordinates.

The non-uniformness of distribution of the represented phenomenon on the initial image means that parts with values of the index greater than average have to be shrunk, parts with values less than average have to be stretched. An electric analog can be represented as an electro-conductive plate with non-constant electrical resistance $R = R(x, y)$ corresponding to values of the regarded index (x and y are Cartesian coordinates on the initial image, i.e. - on the plate). Equipotential lines will change their form and potentials U and V for to ways of applying electric tension to the plate will give a curvilinear system of coordinates. Let us consider the mapping from the initial image (i.e. - from the plate) to the plane with Cartesian coordinates which takes a point of the plate to the point with (Cartesian) coordinates U and V. In parts with lower resistance R the distances between equipotential lines of potentials U and V will be greater than those in part with higher resistance. Thus parts with low resistance will be shrunk by the described mapping, and parts with high resistance will be expanded. Moreover the distance between equipotential lines is inversely proportional to the resistance R. So the coefficient of linear change for the described mapping is directly proportional to the resistance R (at least in directions, orthogonal to equipotential lines), the coefficient of area change is (approximately) directly proportional to the square R^2 of the resistance. It is not exactly proportional to R^2 because the angles between equipotential lines are generally speaking not constant. Therefore if we take the plate with the resistance R directly proportional to the square root of the density of the regarded index, we shall obtain the mapping which makes the density of the regarded index approximately constant, i.e. we shall obtain an approximation to the desired anamorphosis (Fig. 41-b).



Legend: a—transformed image in the rectangular coordinate system
b—the transformation of rectangular coordinates in the heterogeneous space
c—equalization of the gradient of the electric field by a choice of tensions on boundaries of the model
d—equalization of the gradient of the electric field by a change in the configuration of boundaries.

Fig. 41: Model transition from the initial cartographic image to the anamorphated one based on the electric modelling: a — anamorphated image in rectangular coordinate system; b — transformation of rectangular coordinates in the nonhomogeneous space; c — smoothing of the gradient of the electric field by selecting the tension on the model boundaries; d — smoothing of the gradient of the electric field by changing the boundary configuration.

However, as it follows from the same picture, for finding the coordinates which correspond to the distribution of the electric potential, it does not suffice to apply the tension to the edges of the quadrangular plate because the equipotential lines intersect them with different angles. The problem can be reduced to determination of the distribution of the electric potential on the boundaries of the model. This can be achieved by smoothing the gradient of the electric field on homogeneous outlying areas of the model. Practical realization of this problem can be achieved in two ways: either by consecutive changes of the boundaries configuration (Fig. 41-d) or by selection of the tension applied to different parts of the boundary of the model (Fig. 41-?). [1]

Thus, in this case preliminary work consists in creation of an electric model of the distribution of the regarded index. The un-uniformness of the model is achieved by making it of an electroconductive paper of different sorts or by perforating of this paper (or by the combination of these two methods. Futher there is made a levelling of the gradient of the electric field on homogeneous peripheral parts of the model. After that the model is connected with the potentiometer and a current source in such a way that to make it possible to take the coordinates X and Y in turn. the results of the measuring are fixed by a computer. The anamorphosis itself is compiled by the plotter, which draws lines using the obtained coordinates X and Y of points.

More clearly the determination of coordinates in the space of the uniformed phenomenon can be understood in the following way. To the two opposite sides of the plate with non-constant resistance there is applied the electric tension. With the help of special device - electrointegrator (Fig. 42) the positions of lines of constant potential are found. These lines correspond to the (straight) lines with constant value of the (Cartesian) coordinate U on the plane with uniformed phenomenon. After that the tension is applied to the other pair of opposite sides of the plate and the lines of constant potential are determined again. They correspond to the lines with constant value of

[1] *It can be shown that this method can lead to the (exact) anamorphosis if and only if the initial distribution of the index can be made uniform by a conformal transformation.*

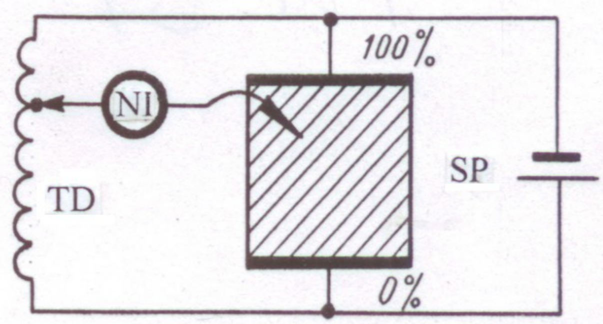


Fig. 42: Block–diagram of the electric integrator: (SP) — source of power suply; (TD) — the tension divisor; (NI) — null–indicator.

the V==coordinate. Now if we transform the net of curves (net of lines of constant potentials) into the orthogonal net of straight lines, we shall obtain the transformation of the whole cartographic image.

As an illustration of the use of the method in the Fig. 29 there is shown the example of transformation of the map of population of Africa into an equidemic projection (Fig. 43). In this case the transformation (anamorphating) has been produced with the help of the model, made of different sorts of the electro-conductive paper (Raspolozhenskiy, Sventek, Tikunov, 1972).

What sort of differences does this method have with respect to other manual methods known earlier? First of all - weak dependence of the result on the executor, and, consequently, a definite objectivity. In the second place there usually is one-to-one correspondence between coordinates on the anamorphosis and on the initial map. However the method of electric simulation suffers from grave shortcomings. It permits to obtain only an approximate solution. Moreover, it is not quite invariant (in particular the result depends on some solutions of the ex-

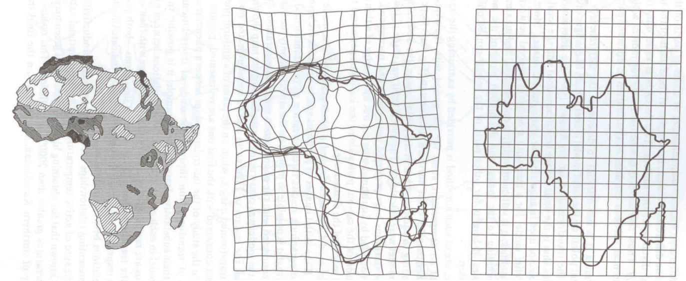


Fig. 43: Anamorphated image of Africa based on the number of population.

ecutor). For example change of the position of the initial image with respect to the rectangular system of coordinates leads to another variant of the anamorphated image. At last one of most labour-consuming parts of the method, namely - the creation of the electric model, remains to be the product of the manual labour.

II.4. Photographic method of compilation of anamorphosises

In the arsenal of geography, there is also a photographic method of construction of anamorphated images. For sure each of you can remember photos on which hands outstretched to the camera become disproportionately large. Used purposefully this effect can help in construction of anamorphosises (Bryukhanov, Tikunov, 1983). Just as the method of electric simulation this method is effective only for creating anamorphosises from images with small variation of the density of phenomenon. It is connected both with technical difficulty of making and using models with big and sharp differences in values of indicators and with the approximate character of the method itself. On the other hand its realization is more simple if compared with the electric simulation.

The essence of the method consists in optical projection of the initial cartographic image on the surface of a relief model, created on the base of the regarded index, with subsequent photographing of the obtained transformed picture. As the result the initial image is reproduced in a central projection with peculiar to it scale distortions, which are made deliberately for obtaining the anamorphosis.

The method can be realized with the use of a device consisting of two optical projectors (like multiplex projectors (projectors of multiplex)). One of them is intended for projecting of the initial image on a relief model, the second one - for photographing the picture reproduced on the model. It is possible to use only one projector if at first the projected image has being drawn on the relief model and then it has being photographed with the use of the same projector. The second way does not demand elimination of distortions caused by necessary, even if small, inclination of one of the projectors.

For computing the heights of individual points of the relief model depending on desirable change of corresponding areas on the initial map (i.e. on the density of the regarded index) it is possible to use the known from aerophotogeodesy dependence between the difference of scales of the aerophoto and the difference of heights of points of the territory:

$$\Delta m = \pm \frac{h}{f}\,. \tag{1}$$

Here Δm is the difference between denominators of scales of parts of the aerophoto, h is the difference between the heights of these parts of the territory, f is the focal distance of the camera. Of cause this formula is valid only for small values of h/f. Because of that this method can be used only for indices with respectively small differences between densities of distribution in different parts of the regarded region.

Using this formula it is not difficult to determine the differences h between heights of different points of the relief model if there are known coefficients of changes of areas for their images on the anamorphosis (they are determined by the values of of the index in the base of the anamorphosis) and the focal distance of the optical projector used for making photos. Moreover for creation of the relief model for a map with constant interval between values, corresponding to isolines, it is enough to define only minimal and maximal values of scales, corresponding to the most low and the most high parts of the model. All intermediate values of heights for other parts of the model can be obtained by simple interpolation.

As an example of the use of the method let us indicate anamorphating of the map of sums of air temperatures for the period with the average daily temperature higher than 10 centigrade for the territory of the Northern Kazakhstan (Fig. 44-A). Depending on the values corresponding to each isoline of the initial image the desired heights of the future relief model were calculated. The model itself was made of foam plastic plates. For it the isolines were projected on plates and corresponding pieces (parts of plates, bounded by isolines) were cut out by the edge of a thin soldering iron. Superposition of the cut out plates along the isolines permits to reproduce the relief model of the desired size. Sharp ledges between plates were smooth out by the white putty. It is also possible to use white plasticine, the powder of gyps and so on.

For obtaining anamorphated images by the offered method, the use of the mountain transformer is possible too. In former times they were

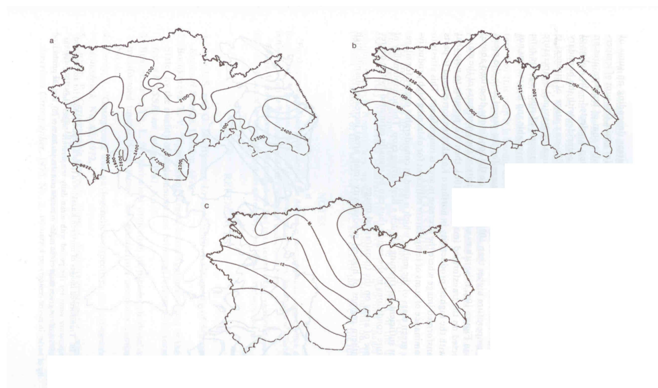


Fig. 44: Initial cartographic image of the North Kazakhstan: A — air temperature sums for the period with the average daily temperature over 10^o C; B — productivity of the green mass of maize in centers/hectare; C — productivity of barley in centers/hectare.

used in photogrammetry for creation and photographing of models of inverse relief of a territory, made directly from enlarged imprints of aeronegatives. Photographing of the model, the shape of

which is taken by the enlarged aerophoto put on it, permits to obtain new negative picture of the territory without distortions because of the relief. The mountain transformer has a mechanism for creation of the desired model with the use of a set of special pivots uniformly distributed over the area, which can be moved out by different heights. This device and the projecting camera of the mountain transformer itself can also be used for direct obtaining of anamorphated images too.

After the relief model has been created by either of methods there is nothing for it but to make the photo of it from the required height. If there are shown initial images of isolines and boundaries on the model, then photographing permits to obtain desired anamorphotic image. Let us note that it is possible to project on the model not only the initial image, but also another one connected with the regarded phenomenon. Thus in the discussed example the model has been made on the base of the map shown on Fig. 44-D, and after that the images of the Fig. 44-A and 44-C were projected on it. Such compilation of anamorphosises, when on the plane with a uniformed phenomenon other phenomena connected with the first one are represented, permits to use it for study of interrelations between phenomena.

The process of anamorphating for study of interrelations must be determined by their meaningful features. For example, it is possible to choose such a height of the model for parts of the territory where relations between phenomena become apparent in the most clear form, that their areas are not changed on the anamorphosis. In this case all deviations both in the direction of decrease and in the direction of increase of areas will show the character of deviations in relations between phenomena and the reasons of these deviations will be different. Therefore for each culture it is possible to represent parts of the territory, where the sum of active temperatures (or other ecological indicators) is mostly favourable for its cultivation, without transformation of areas. In this case for each culture, having its own optimal sum of temperatures, special anamorphosis ought to be compiled and used.

Thus for representing relations between the productivity of green mass of the maize (Fig. 44-B) and the sum of active temperatures (Fig. 44-A) their optimal value ought to be chosen. It is known that for obtaining high productivity of green mass of the maize the sum of temperatures greater than 2000° is sufficient. It takes place on the whole territory of the North Kazakhstan. In this case on the final anamorphated image all parts of the territory will increase their areas depending on sums of active temperatures. However for cultivating maize for the grain this sum of temperatures is not sufficient. Therefore as the favourable sum of temperatures let us choose its average value for the territory of the North Kazakhstan, the isoline of which lies somewhat to the South or approximately coincides with the boundary, beyond which ripening of corn-cons up to the milky-wax stage of ripeness is possible. Since in the most South parts of the territory with the maximal sum of temperatures the aridness of the climate adversely affects the process of ripening, the zone with average

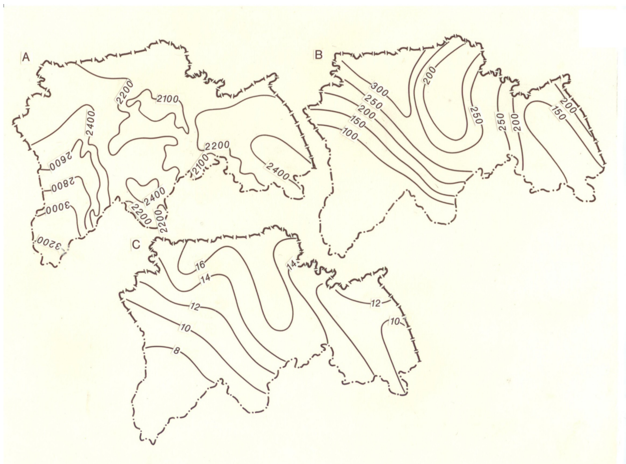


Fig. 45: Images anamorphated by the photographic method: A — an anamorphosis based on air temperature sums for the period with the average daily temperature over 10^o C; B — productivity of the green mass of maize on the anamorphosis; C — productivity of barley on the anamorphosis.

temperatures was regarded as the most favourable and its area has not been changed on the anamorphosis.

The anamorphosis of sums of air temperatures for the period with the average daily temperature higher than 10° centigrade, based on the base of the described conditions, is shown on the Fig. 45-A. If isolines of productivity of the green mass of the maize have been projected on the anamorphated image (Fig. 45-B), then it is possible to note that regions with the highest for the North Kazakhstan productivity correspond to parts of the anamorphosis which decreases their areas insignificantly. It testifies for predominance of sums of temperatures sufficient for obtaining high productivity of the green mass of the maize, which is used in the considerable degree for silo (silage), but is not sufficient for ripening of such a heat-loving culture, as the maize is, for the grain. The non-optimality of temperature conditions in the region can be visually estimated by the degree of decrease of areas on the anamorphosis.

In South regions with the highest degree of increase of areas, in spite of high sums of temperatures, the decrease of productivity takes place. As it has been noted, this fact is connected with the aridness of the climate. Therefore in this case and in subsequent ones it must not be forgotten, that the productivity is determined far from only by the sum of temperatures.

On Fig. 45-C, where isolines of productivity of barley are drawn on the anamorphosis, based on sums of temperatures, the regions with heightened productivity turn out to be connected with the regions, which decrease their areas. But in this case even minimal for North Kazakhstan sums of temperatures are sufficient for growing of such a cold-resistant culture, as the barley is, and its productivity depends on other factors. For this culture it is hardly advisably to determine the relation between the sum of temperatures and the productivity of the barley for the whole regarded territory, as it was done for heat-loving maize. It testifies for necessity to watch closely to correspondence of relations between geographical phenomena when constructing an anamorphosis, based on one of them.

The drawn conclusions are rather simple and only illustrate known regularities. However, in spite of the sketchness of the discussed examples with adequate and inadequate representations of relations with the help of anamorphosises, they permit to understand perspective ways of use of anamorphosises for problems of such a sort.

The discussed methods of analogic compilation of anamorphosises have restrictions, described above. Moreover, they require the use of special equipment, which unlike the computer does not have a general-purpose character. Therefore it is natural that the most attention in this book will be given to numerical methods of compilation of anamorphated images.

Chapter III. Numerical methods of compilation of anamorphosises.

III.1. Mathematical formulation of the problem of compilation of anamorphosises

Numerical methods of compilation of anamorphosises are intended mainly for their realization with a computer. The first problem which arises if we want co compile an anamorphosis consists in the method of description of the initial data, i.e. of the initial cartographic image and of the density of distribution of the index under consideration. There is a number of methods of dealing with cartographic images with the use of computers. They are connected with the general problem of processing of cartographic images and can be regarded as well elaborated (if compared with the problem of compilation of anamorphotic images).

Since it is difficult to hope to obtain a transformation which produces a desired anamorphosis explicitly in one step, it is natural to try to create iterative algorithms which take into consideration on each step the deviation of the density from the constant one on the whole cartographic image (or from the required constant in one of the cells) and correct the density in a suitable way.

For convenience of the further description we shall suppose that the initial cartographic image has been drawn on the plane $\mathbb{R}^2$ with

Cartesian coordinates (x, y). Moreover we shall assume that the projection of the initial cartographic image, from which the anamorphosis has to be produced, is authalic. It means that we shall not take into account possible initial distortions of areas and distances. They can be taken into consideration, but it will make the explanations more complicated, adding nothing essential.

An area anamorphosis must make uniform the density of an index, which has to be defined (described). All known to the authors algorithms of compilation of anamorphosises with the use of a computer proceed from the description of the density based on a division of the territory into parts (cells). It means that for each of these sells the total value of the index is determined and the density is assumed to be constant inside it. The choice of these cells can be different. In some cases the role of them can play different simple geometrical figures (triangles, squares, an so on), which a priori are not connected with existent divisions of the territory (for example, administrative-territorial). In particular they can be organezed into a regular partitioning of the plane. In other cases their role can be played by units of existent divisions (such as administrative-territorial). Sometimes similar algorithms can be used in both cases. Distinctions usually consist in details of their realization in computer programs (for example, in determination of boundaries of cells, in choice of their centers and so on).

An alternative to such an approach to the description of the density can be its definition by an analytic expression (for example, with the use of a trend of some order). In this case it must be possible to try to find the anamorphosis in a form of an analytic expression too. Such an approach has its pluses and minuses but has not been realized yet.

Let $\mathfrak{D}$ be a region of the plane $\mathbb{R}^2$ (the cartographic image of the territory), from which the anamorphosis has to be constructed. Irrespective of the method of describing the density of the regarded index, for general formulation of the problem we can assume that the distribution of the index is determined by a density function $p(z)$, defined apriori on the part $\mathfrak{D}$ of the plane ($z = (x, y)$ is a point of the plane $\mathbb{R}^2$). Without any loss of generality we can assume that the density function $p(z)$ is defined on the whole plane $\mathbb{R}^2$. To make it possible we can define it to be equal to a constant p outside the region $\mathfrak{D}$. For example one can take p equal to the average value $\bar{p}$ of the density function $p(z)$ over the region $\mathfrak{D}$.

An anamorphosis as a transformation of the initial cartographic image can be defined by a map $h : \mathbb{R}^2 \to \mathbb{R}^2$ from the plane to the plane ($h : (x, y) \mapsto (u, v)$) or by two functions of two variables $U(x, y)$ and $V(x, y)$:

$$u = U(x, y), \qquad v = V(x, y). \tag{2}$$

This transformation has to be continuous and one-to-one. It is natural to demand it to be differentiable (at least almost everywhere).

To formulate the condition that the transformation h makes the density $p(x, y)$ uniform, let us remind that the coefficient of change of area by the transformation h near a point (x, y) is equal just to the value of the Jacobian

$$J = J(U, V) = \frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y} - \frac{\partial U}{\partial y} \cdot \frac{\partial V}{\partial x} \tag{3}$$

of the transformation h at this point. Therefore the condition that the transformation 2 makes the density $p(x, y)$ uniform (and equal to $\bar{p}$) can be written as $J(U, V) = p(x, y)/\bar{p}$. Therefore the problem of finding an anamorphosis reduces to the problem of finding a solution of the equation

$$\frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y} - \frac{\partial U}{\partial y} \cdot \frac{\partial V}{\partial x} p(x, y)/\bar{p} \tag{4}$$

(for which in addition the pair $[U(x, y), V(x, y)]$ defines a one-to-one transformation).

Compilation of anamorphosises with the help of a computer meets difficulties mostly of two types. First of them are connected with the computer realization of the algorithm. These are problems of conversion of the initial cartographic image into a form suitable for the computer handling, of a numerical realization of the algorithm with a control of a preservation of the uniqueness (one-to-one) of the transformed cartographic image, of the representation of results in a form of the cartographic image. These difficulties have their own peculiarities for different algorithms and we shall discuss them briefly while considering existing approaches.

The second class of difficulties is connected with the problem which is of particular significance. It should be kept in mind, in particular, if we want to compare anamorphosises obtained with the use of different methods. The matter is that the condition of making the given density uniform does not determine the anamorphosis in a unique way. There exist an infinite set of transformations which satisfy this condition. It can be easily understood from the fact that for two unknown functions $U(x, y)$ and $V(x, y)$ which define the anamorphosis there is only one equation (4). Already obtained anamorphotic image without destruction of the constancy of the density can be changed by the application of any transformation which preserves areas, for example: 1) $(u, v) \mapsto (k \cdot u, k^{-1} \cdot v)$ (the expansion along one of axes and the compression along the other one with one and the same coefficient); 2) $(u, v) \mapsto (u + f(v), v)$ (translations of horizontal straight lines along themselves on different distances); 3) $(u, v) \mapsto (u, v + g(u))$ and so on.

This arises the problem how to choose among different such transformation that one (or those) which ought to be regarded as suitable. If we shall assume that the only our aim is to find any transformation which makes the density $p(x, y)$ uniform, then the problem of construction of the anamorphosis can be solved rather simply. For example, we can take the transformation

$$u = \int\limits_0^x p(x', y)dx', \qquad v = y. \tag{5}$$

However it is not difficult to realize that it is not the one which we want to have for the role of the anamorphosis. This transformation preserves all points on the vertical coordinate axis, preserves straight lines parallel to the horizontal coordinate axis, i.e. it deforms the territory only in the horizontal direction. So it is not invariant with respect to the choice of the system of coordinates. It is natural to demand such an invariance for the method of anamorphating. At least one should demand the invariance with respect to translations of the origin $x \mapsto x + a$, $y \mapsto y + b$ (if we shall assume that on the surface of the earth there are two distinguished directions: south - north and west - east). But as an ideal the invariance with respect to any choice of the Cartesian coordinate system has to be demanded.

As an additional requirement which can fix the choice of the anamorphosis we can try to use the condition of conformality of the transformation 2. The conformality of a transformations means that locally it changes all distances by multiplying on one and the same coefficient non-dependently on the direction or, what is just the same, preserves angles between curves. If there exists such a transformation which locally changes all distances by multiplying on $\sqrt{p(x,y)}$, then it ideally fits on the role of the anamorphosis. In particular in this case it is unique (up to an isometry of the plane) and it does not depend on the choice of the coordinate system. In addition it can be used for solving of a number of problems different from the problem of finding an anamorphosis for the given density. For example such transformation can be used for finding a solution of the Bunge's problem about the construction of a system of centers for which the sum of distances from all the residents distributed over the territory is minimal (see, for example, Gusein-Zade, 1982).

To a regret a conformal transformation with the given coefficient of expansion $\sqrt{p(x,y)}$ (or with the given coefficient of the change of areas, equal to $p(x,y)$) does not always exist. This can be easily understood by comparison of the number of unknown functions and the number of equations which define such a transformation. The condition of conformality of the transformation 2 with the coefficient of expansion equal to $\sqrt{p(x,y)}$ can be written as the following system of equations:

$$\begin{aligned} \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 &= p(x,y), \\ \left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 &= p(x,y), \\ \frac{\partial U}{\partial x} \cdot \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \cdot \frac{\partial U}{\partial y} &= 0. \end{aligned} \tag{6}$$

Thus this condition consists of three equations for two unknown functions $U(x,y)$ and $V(x,y)$. As usual such system has no solutions. For existence of such transformation it is necessary that the density function $p(x, y)$ satisfies the equation

$$\Delta\, ln\, p \equiv \frac{\partial}{\partial x}\left(\frac{\partial p/\partial x}{p}\right) + \frac{\partial}{\partial y}\left(\frac{\partial p/\partial y}{p}\right) = 0 \qquad (7)$$

($\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian). This equation means that the curvature defined by the metric $p(x, y) \cdot (dx^2 + dy^2)$ must be equal to zero. Generally speaking it is possible to find a transformation with indicated properties if we shall admit that it transforms the flat cartographic image into an image on a curved surface. However it can not be regarded as the aim of the discussed problem. In addition

a compilation of such an image and handling of it as minimum are not easy. W.Tobler (Tobler,1979b, 1986) offered to choose the transformation which changes areas in a proper way and is as close to a conformal one as possible. However it is not clear how to achieve this in practice.

III.2. Tobler's algorithm

One of the first numerical method of compilation of anamorphosises was offered by W.Tobler. In this algorithm it is supposed that the territory is divided into

quadrangular cells (initially into rectangles). For each quadrangle we choose such displacements of vertices which lead it to the desired area. Moreover the vectors of displacement are chosen in such a way that they are parallel to the bisektrices of the coordinate quadrants that is they are of the form (a, a) or $(a, -a)$. The last condition serves for the unique determination of the vectors of displacement. If vertices of one of the quadrangles (say Q) have coordinates (x_i, y_i) $(1 \leq i \leq 4)$ and their vectors of displacement are equal to (Δ, Δ), $(-\Delta, \Delta)$, $(-\Delta, -\Delta)$, $(\Delta, -\Delta)$ for $i = 1, 2, 3, 4$ respectively, then it can be shown that Δ can de determined from the equation

$$4 \cdot \Delta^2 + \Delta \cdot (x_1 + x_2 + x_3 + x_4 + y_1 + y_2 + y_3 + y_4) + A - A' = 0, \quad (8)$$

where A is the actual area of the quadrangle Q, A' is the desired area of it ($A' = A \cdot p_Q/\bar{p}$, where p_Q is the density of the regarded index in the quadrangle Q, $\bar{p}$ is the average density of the index on the whole cartographic image). For each vertex of the lattice the vector of displacement on each iterative step is determined as the sum of four vectors corresponding to cells (quadrangles) adjacent to it). In accordance with the determined vectors we carry out the displacement of all vertices. The absence of breaches of integrity of the cartographic image (as overlapping of neighboring cells by each other) should be controlled. After that new areas of cells are calculated and the iterative step is repeated. The process finishes if the deviation of the obtained density of the distribution of the regarded index from a constant one becomes small enough.

An advantage of this algorithm consists in its simplicity and in absence of priorities of some cells over others (for example, in the order of choice of them). But it has evident shortcomings too. First of all the result depends considerably on the choice of directions of the coordinate axes. It is not connected essentially with the fact that initially we take a division of the territory into rectangles with sides parallel to the coordinate axes (in principle the algorithm admits any division into quadrangles). The main reason is in the choice of vectors of displacement of vertices for each quadrangle parallel to the bisektrices of the coordinate quadrants which is necessary for the uniqueness of determination of them. Secondly on each step the vector of the displacement of a vertex is determined only by four quadrangles adjacent to it. Therefore a presence of, for example, a part with high density on the territory will be "felt" by a vertex (in the sense of its displacement) only after the number of iterative steps equal to the number of cells between the vertex and the indicated part. So this iterative process cannot converge very quickly. Moreover for steps after the first one (when quadrangles are not rectangles already) the equation 8 can have no solutions if the desired area A' is small enough , and it should be explained what to do in such a case. At last it is not evident (and it has not be proved) that this algorithm converges for any initial distribution of the density (even if there are no problems with solvability of the equation 8).

A development of the described method (due to W.Tobler as well) uses not a regular net of quadrangles, but any division of the territory into polygons. To each polygon there correspond vectors, associated

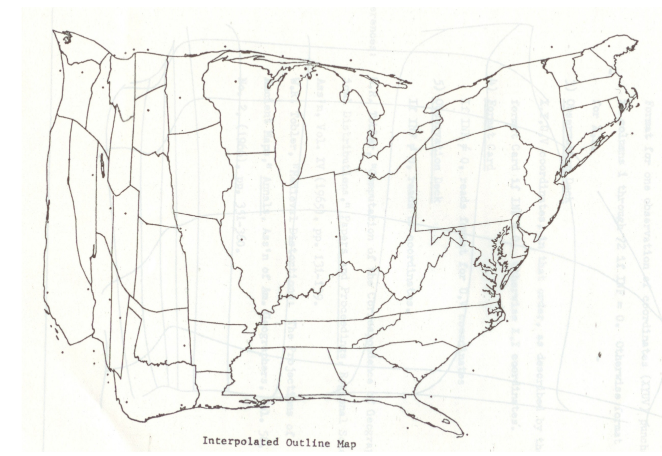


Fig. 46: The anamorphosis of the territory of the USA, based on the population of states and compiled with the use of the algorithm of Tobler.

to its vertices in such a way that the translation of vertices with the help of them leads this polygon to a similar one with the desired area. On each step the resulting displacement of each vertex is defined as the sum of vectors corresponding to adjacent polygons. Details of realization of the algorithm are not completely clear (at least for the authors). For example, some vertices may have two adjacent polygons, while some others — three or more. It is not clear if in all cases it is possible to define the displacement vector as the sumw of the corresponding vectors: if it will not lead to breaches of integrity of the cartographic image.

As an example of use of the improved version of the described algorithm let us indicate the anamorphated image of the USA, published as a post-card (Fig. 46).

An algorithm, based on a development of the ideas of W.Tobler can be found in the papers (Tikunov, 1986, 1988). The essence of it is the following. The territory, which will be anamorphated, is covered by a regular net, for instance, by a net of quadrangles. The index, used as the base of the anamorphosis, is computed for all cells of the net. The sizes of the cells are determined by the detailes of initial data and are chosen with the help of an experiment, by approximation of ctatistical surfaces of initial indices for different sizes of cells with subsequent comparing of approximate surfaces with real ones. The net should cover the territory to be anamorphated and adjacent regions in such a way that all the data can be represented in the form of a rectangular matrix, convenient for processing with the help of a computer. Let us note that values of indices for cells, covering adjacent regions, are taken to be equal the average value $\bar{p}$, computed for the territory under consideration itself.

The role of vertices in the algorithm play both the vertices of the cells (squares) and the middles of their sides. Later on these cells are deformed into, generally speaking, irregular octagons. It is assumed that under the influence of the cell with the number i, which has the area equal to s_i, each vertex is displaced on one and the same distance in the direction of the straight line, which connects the vertex with the center of the cell. This distance d_i is computed by the formula

$$d_i = \frac{1}{\omega \cdot \sqrt{\bar{P}}} \left(\sqrt{\frac{P_i}{s_i}} - 1 \right), \tag{9}$$

where P_i is the total value of the index, on which the anamorphosis is based, for the cell i (the density of the considered index in the cell i is equal to P_i/s_i), $\bar{P}$ is the average value of the index per one cell (P_i); $\bar{P}$ do not change in the process of the iterations, s_i do change), ω is a correction factor which is determined by the character of initial data (for smooth variations of the index we take $\omega = 1$, for more sharp ones one takes $\omega = 2$, and so on). So in the method offered in the indicated paper the influence of a cell on vertices does not diminish with the increase of the distance from it. The displacement of a vertix is defined as the (vector) sum of its displacements, corresponding to all cells. The described displacement of all the vertices is one iterative step of the algorithm. This step is repeated untill the density of the regarded index is close enough to the constant one.

This algorithm (as the majority of other ones as well) permits to include into the set of vertices any other points of the initial image, for instance - points of boundaries of administrative- territorial units. These additional vertices are displaced on each iterative step, but they are not taken into account for calculation of areas of cells and therefore of displacements of other points. It gives possibility to draw administrative boundaries on the final anamorphosis (for example with the help of the plotter).

This method is illustrated with the help of an example, which shows the possibility to use anamorphosises for the analysis of distribution of geographical objects in an artificially uniformed demographic space. In connection with this problem there is an interesting anamorphosis compiled manually by A.Getis (Getis, 1963). He analyzed the distribution of department stores for one of the American cities. If we shall ignore the subjectivity of compilation, it is possible to say, that it is most profitable to create new stores in places where their density on the anamorphosis is minimal. Moreover, if the objective is to achieve maximal profit, then the net ought to be constructed in the space of the income (just how it was done by the author), but if the objective is convenience of service to all groups of population,

independently of their parses, then the construction ought to be executed in the demographic space (Tikunov, Pipkin, 1990). The description of this experiment illustrates the regarded method with the use of the example of evaluation of the net of food stores in Troy, New York.

Troy is an old industrial center at the confluence of the Mohawk and Hudson rivers. It is part of the Albany-Schenectady-Troy capital city metropolitan area with an aggregate population of 795,019 (according to the last U.S. population census of 1980). In 1980 the city of Troy itself had a population of 56,638. The city's industrial base has declined over recent decades, but nevertheless the share of manufacturing in the non-military labor force is 18.56.9unemployment also is high - 8.96.5Review, December 1986, p.19) and change the economic base of urban development. In 1986 there were only a few major employers - two colleges, three hospitals, the city and county administration, and one major industrial firm. Within the city there are 54 food stores, which are represented by dots on the street plan of Troy (Fig. 47). The latter were the subject of the research performed in (Tikunov, Pipkin, 1990).

The aim was to determine whether there was a correlation between

Fig. 47: Arrangement of streets and food shops in Troy.

the distribution of food stores and population density within the city limits. For it an anamorphosis of the city has been compiled First of all the population density (according to the census of 1980) was calculated within the limits of the rectangular grid cells shown in Fig. 47 in the form of grid tick marks. The anamorphosis compiled on the base of these data is depicted in Fig. 48. As might be expected, the nature of location of food stores in an equidemic projection is more regular than on the actual earth's surface. However, even here it is possible to find deviations which must be analyzed in the development of the network of stores, if the objective is convenience of service to the entire population.

III.3. Compilation of polyfocal anamorphosises

The method of polyfocal projections was offered in (Kadmon, Shlomi, 1978). In accordance to the understanding used in the book they can be called polyfocal anamorphosises. Just as the previous method (and in contrast to the majority of methods, described in the book) it is not intended for making uniform any specific density. The aim of the method is to enlarge the scale near several chosen centers.

If we have only one such center it can be done in the following way.

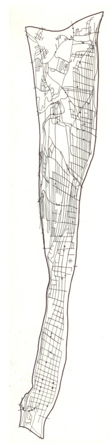


Fig. 48: Arrangement of food shops in Troy on the background of the anamorphosis compilated on the base of the number of population

We can take the polar system of coordinates (r, φ) with the origin at the distinguished point and consider the transformation which takes a point (r, φ) to the point $(r \cdot (1 + f), \varphi)$. Here $f = f(r, t, u, \ldots)$ is a function on the distance r and additional parameters t, u, $\ldots$, influencing scale/distance relation, such as time and cost. For enlarging the scale near the origin the authors suggest to use the function f of the form

$$f(r, t, u, \ldots) \frac{A}{1 + C \cdot r^{(d_0 + d_1 t + d_2 t^2 + \ldots + e_1 u + e_2 u^2 + \ldots)}}. \tag{10}$$

. Actually they used only the case $d_0 = 2$, $d_i = 0$, $e_i = 0$, $\ldots (i = 1, 2, \ldots)$, i.e. they did not consider influence of any additional parameters. Thus the function $f = f(r)$ was of the form $\frac{A}{1+C \cdot r^2}$.

In other words (in Cartesian system of coordinates in which the chosen center has coordinates (x_1, y_1)) the used transformation can be described by the formula

$$(x, y) \mapsto (x + \Delta x_1, y + \Delta y_1), \tag{11}$$

where $r = \sqrt{(x - x_1)^2 + (y - y_1)^2}$, $\Delta x_1 = (x - x_1) \cdot f(r)$, $\Delta y_1 = (y - y_1) \cdot f(r)$.. Thus for any point (x, y) of the plane the vector of displacement is equal to $(\Delta x_1, \Delta y_1)$.

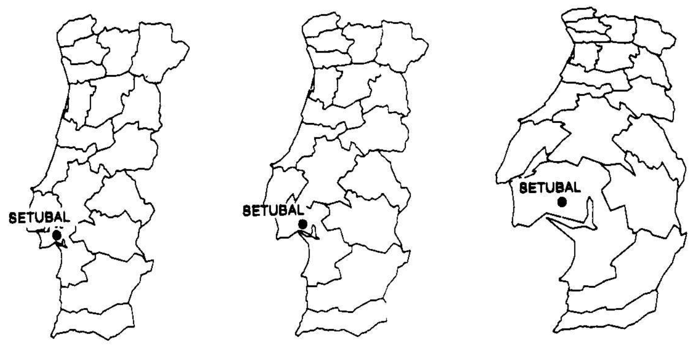


Fig. 49: Unifocal projection for the territory of Portugal with respect to Setubal.

An image obtained this way is shown in Fig. 49 (Cauvin, 1885).

If we have several centers (x_1, y_1), (x_2, y_2), ..., (x_n, y_n), we can consider the displacement of a point equal to the (vector) sum of displacements, corresponding to all the centers. So the polyfocal projection is defined by the transformation

$$(x, y) \mapsto (x + \Delta x_1 + \Delta x_2 + \ldots + \Delta x_n, y + \Delta y_1 + \Delta y_2 + \ldots + \Delta y_n)\,, \quad (12)$$

where $(\Delta x_i, \Delta y_i)$ $(i = 1, 2, \ldots, n)$ are determined by the indicated formula for the point (x_i, y_i). Moreover the coefficients A and C can be taken different for different centers, i.e.

$$\Delta x_i = (x - x_i) \cdot \frac{A_i}{1 + C_i \cdot r_i^2}, \qquad \Delta y_i = (y - y_i) \cdot \frac{A_i}{1 + C_i \cdot r_i^2}\,. \quad (13)$$

Use of different values of the coefficients A_i and C_i permits to have different scales near different centers. The choice of these coefficients is not formalized and therefore is an element of art.

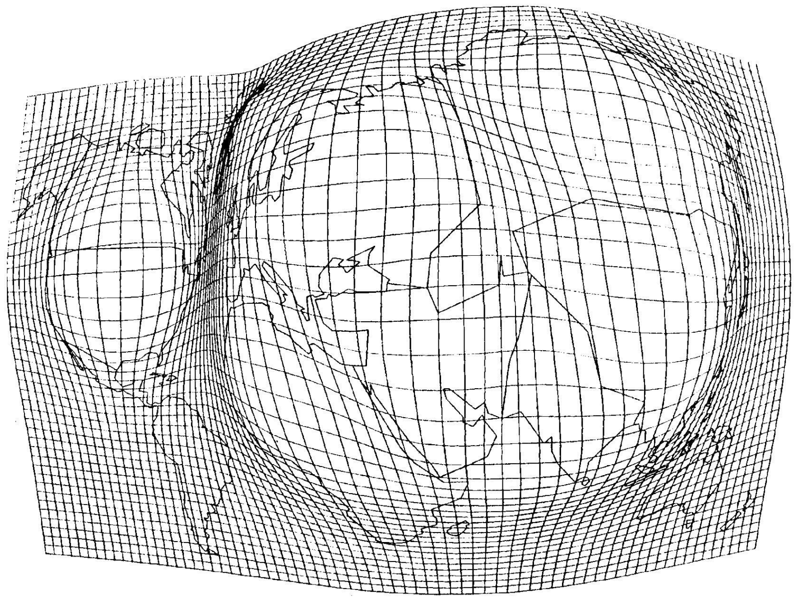


Fig. 50: A polyfocal anamorphosis of the World.

An example of graphic image obtained by the polyfocal projection from (Kadmon, Shlomi, 1978) is shown on the Fig. 50.

III.4. Method of triangles

This method was elaborated in the Moscow State University in the early eighties (see (Petrov, Serbenyuk, Tikunov, 1983)). As the method of W.Tobler it deals not with a natural division of the territory but rather with an artificial one. The territory (or its cartographic image) is supposed to be triangulated i.e. covered by a set of triangles in such a way that different triangles do not intersect each other and can have either one common side or one common apex. Moreover, it is desirable that the system of triangles is chosen in such a way that the territorial units are approximated by unions of some of triangles. For each of the triangles the density P_i of distribution of the considered index is computed. One has to choose an indicator f, which is defined for each vertex of the net of triangles and which characterizes differences in densities of distribution of the phenomenon under consideration for the triangles adjacent to this vertex. This means that the value f_j of the indicator f for a vertex j is always defined, is non-negative and is equal to zero if and only if densities $P_{i_j(1)}$, $P_{i_j(2)}$, $\ldots$, $P_{i_j(m_j)}$ of the index under consideration in all the triangles adjacent to this vertex are equal to each other. Here m_j is the number of triangles adjacent to the vertex j, $i_j(1)$, $i_j(2)$, $\ldots$, $i_j(m_j)$ are their numbers. The role of this indicator can be played by one of the following:

$$f^{(1)} = \sum_{\ell=1}^{m_j} (P_{i_j(\ell)} - \bar{P}_j)^2,$$

$$f^{(2)} = \sum_{\ell=1}^{m_j} |P_{i_j(\ell)} - \bar{P}_j|,$$

$$f^{(3)} = \max_{1 \le \ell < k \le m_j} |P_{i_j(\ell)} - P_{i_j(k)}|\,, \tag{14}$$

where $\bar{P}_j = \left(\sum\limits_{\ell=1}^{m_j} S_{i_j(\ell)} \cdot P_{i_j(\ell)} \right) / \left(\sum\limits_{\ell=1}^{m_j} S_{i_j(\ell)} \right)$, S_i is the area of the i-th triangle.

For numerical realization of the method the fist formula was used. After that an arbitrary vertex j of the net of triangles is chosen. There is determined a region of the plane, which contains the regarded vertex, and such that a displacement of the vertex into any of its points does not break the property of the image to be one-to-one, i.e. does not break the mutual arrangement of the adjacent triangles. For the realization of the algorithm this region is chosen as a circle, the radius of which is determined by a special formula. Within the circle there is chosen a new position of the vertex, for which the value of the indicator f is minimal as far as possible. It is offered to determine such a position with the help of the Monte Carlo method, i.e. of the method of random search. After the change of the position of the vertex into the found point new densities of distribution of the regarded index for all the adjacent triangles are calculated (These densities are equal to $P_{i_j(\ell)} \cdot S_{i_j(\ell)} / \widetilde{S}_{i_j(\ell)}$, where $\widetilde{S}_{i_j(\ell)}$ is the new area of the corresponding triangle. After that the procedure is repeated for next vertex and so

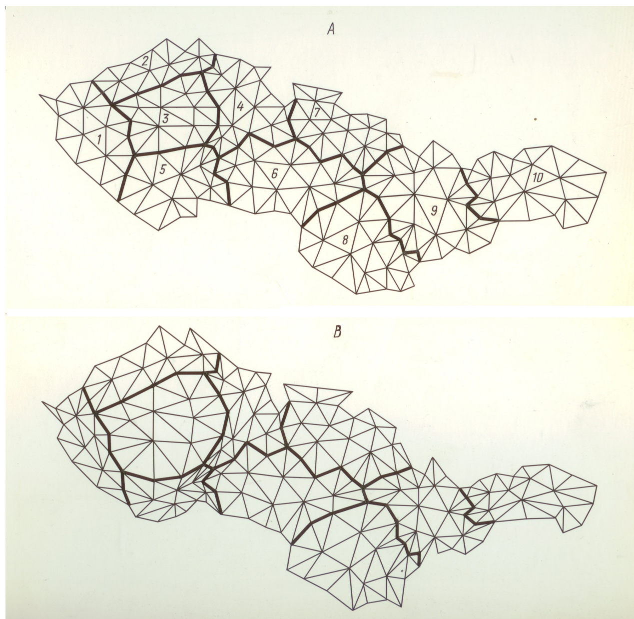


Fig. 51: Initial generalised image of the distribution of population over the territory of Czechoslovakia (A) and its anamorphosis (B).

on until the difference in densities of distribution of the regarded index is less than a chosen in advance value.

Obvious advantages of the method are: simplicity, independence on choice of any coordinate system, preservation of topological likeness to the

original. The main shortcoming of the method consists in essential dependence of the result on random choices, which take place in it: the order of processing of vertices, the random search of new position of the chosen vertex.

A result of application of the method to a map of characterizing space changes in the population distribution (Fig. 51-A) is shown in the Fig. 51-B.

III.5. Algorithm of the Lawrence Berkeley Laboratory

A detailed description of an algorithm of compilation of anamorphotic images is contained in the paper (Selvin et al, 1984). Obtained anamorphosises (density-equalized maps in their terminology) were used for displaying epidemiologic data (mainly for the territory of San Francisco).

Let $\bar{p}$ be the average density of the considered index over the whole territory. On each step of the algorithm one of the cells (say A) under consideration is taken. Let c be the density of the considered index in the cell A and let C_A be the center of masses of this cell. This center can lie outside the cell A itself if this cell has an intricate form (non-convex or non-simply connected). After that there is determined the deformation h_A of the cartographic image which preserves the center C_A, changes the area of the cell A to the required one, and does not change the areas of all other cells.

In order to describe this deformation let us take a system of polar coordinates (r, θ) with the origin at the point C_A. $Q_i = (r_i, \theta_i)$ be one of the points used for digitizing the cartographic image (a priori it can be an arbitrary point of the plane). Usually it is one of the points of boundaries of cells (territorial units) under consideration and lies outside of A, but this is not necessary. In particular it can be any distinctive point on the map (for example corresponding to a big city). Let us regard the straight line connecting the points C_A and Q_i and let $Q_{i,j} = (r_{i,j}, \theta_i)$ $(j = 1, 2, \ldots, n;\ r_{i,1} < r_{i,2} < \ldots < r_{i,n})$ be the points of intersection of this straight line with the boundary of the cell A. In the simplest case (the cell A has a simple form, for example it is convex, the point P_i lies outside A) there is only one point of intersection $(n = 1)$. Now the image $\widetilde{Q}_i$ of the point Q_i under the deformation h_A is determined to be $(\widetilde{r}_i, \theta_i)$ were $\widetilde{r}_i$ is equal to $f \cdot r_i$, $f = (1 + (M_A - 1) \cdot R_i^2 / r_i^2)$, $M_A = p_A / \bar{p}$, the definition of R_i (or rather R_i^2) depends on the geometry of the region A and on the position of the point Q_j. If the center C_A is inside A, and the point Q_i is outside of A (and hence n is odd), then

$$R_i^2 = r_{i,1}^2 - r_{i,2}^2 + r_{i,3}^2 - \ldots + r_{i,n}^2\,; \tag{15}$$

if both the center C_A and the point Q_i are inside A (n is even), then

$$R_i^2 = r_{i,1}^2 - r_{i,2}^2 + r_{i,3}^2 - \ldots - r_{i,n}^2 + r_i^2\,; \tag{16}$$

if both the center C_A and the point Q_i are outside of A (n is even), then

$$R_i^2 = -r_{i,1}^2 + r_{i,2}^2 - r_{i,3}^2 + \ldots + r_{i,n}^2\,; \tag{17}$$

if the center C_A is outside of A, and the point Q_i is inside A (n is odd), then

$$R_i^2 = -r_{i,1}^2 + r_{i,2}^2 - r_{i,3}^2 + \ldots - r_{i,n}^2 + r_i^2\,. \tag{18}$$

This deformation changes the shape of the chosen cell rather slightly (if the cell is convex, the deformation acts on it simply as a dilatation). The shapes of other cells change more significantly. This operation is repeated for a second cell, for a third one and so on up to the last one. In the result we obtain a deformation which (approximately) possesses the desired properties. The obtained deformation does not make the density exactly constant because the straight line segments of boundaries of cells must be transformed into curves, but are regarded approximately straight themselves. Transformations corresponding to areas which do not belong to the regarded territory but are inside it (such as lakes) depend on form in which we wish to see them. They may be shrunk by taking $M_A = 0$ for them, or their areas may be preserved by taking $M_A = 1$.

The described method permits to obtain anamorphosises of rather good quality. The realization of it is respectively simple. The only difficult task is calculation of points of intersection of straight lines connecting the center C_A of the cell A and points Q_i with the boundary of the cell A.

The main shortcoming of this method consists in the fact that the final result depends essentially on the order in which the cells are taken. Another problem arises if the straight line connecting the center C_A of the cell A and a point Q_i contains one of segments of the boundary of A (or if such a segment is very close to the indicated line). It is possible especially on the first step of the algorithm because the precision of the initial data is determined in the process of digitizing and usually is not very high. In this case the described transformation is not continuous and we lose the property of continuity of the cartographic image. Besides that the result is obtained with the use of a large number of steps equal to the number of the regarded cells. Moreover on each step only the area of one of the cells has been corrected. So the errors admitted on each step (for example the errors of calculations, the errors of the substitution of curves, into which segments are transformed, by segments or polygonal lines ...) are not corrected later and therefore are accumulated.

A reesult of application of the described algorithm to the territory of the United States with the density of population as the index in the base of the anamorphosis is shown on Fig. 52. Results of application to the territory of San Francisco with the density of distribution of white and black male population considered on it are shown on Fig. 53 (the initial image), Fig. 54 and 55.

III.6. Algorithm of Dougenik J.A. et al.

¿From the discussions of the methods described above some directions of improvement of algorithms follow. First of all it is desirable that on each iterative step all cells of the division exert their influence on the displacement of points (in particular - vertices). For it we can assume that the displacement of a point is equal to the (vector) sum of displacements originated from the influence of separate cells. Secondly it is natural to assume that the influence of a cell on a point consists in a displacement of this point along the straight line connecting the point with the cell (more precisely with a point defined by it, for example, with its center of masses). This condition is connected with the requirement of the invariance of the algorithm with respect to a choice of the coordinate system. And at last it is natural to expect that the displacement of a point under the influence of a cell decreases with the growth of the distance from this cell.

It seems that first such algorithm has been offered in (Dougenik, Chrisman, Niemeyer, 1985). In order to describe the method let us regard a situation when on the infinite territory there is a bounded part (for simplicity we can assume that it is a circle) on which the density of the distribution of the index is different from the average density (on the remaining part of the territory the density coincides with the average one). It is easy to understand that for the (natural) anamorphating the displacements of points will be less for those of

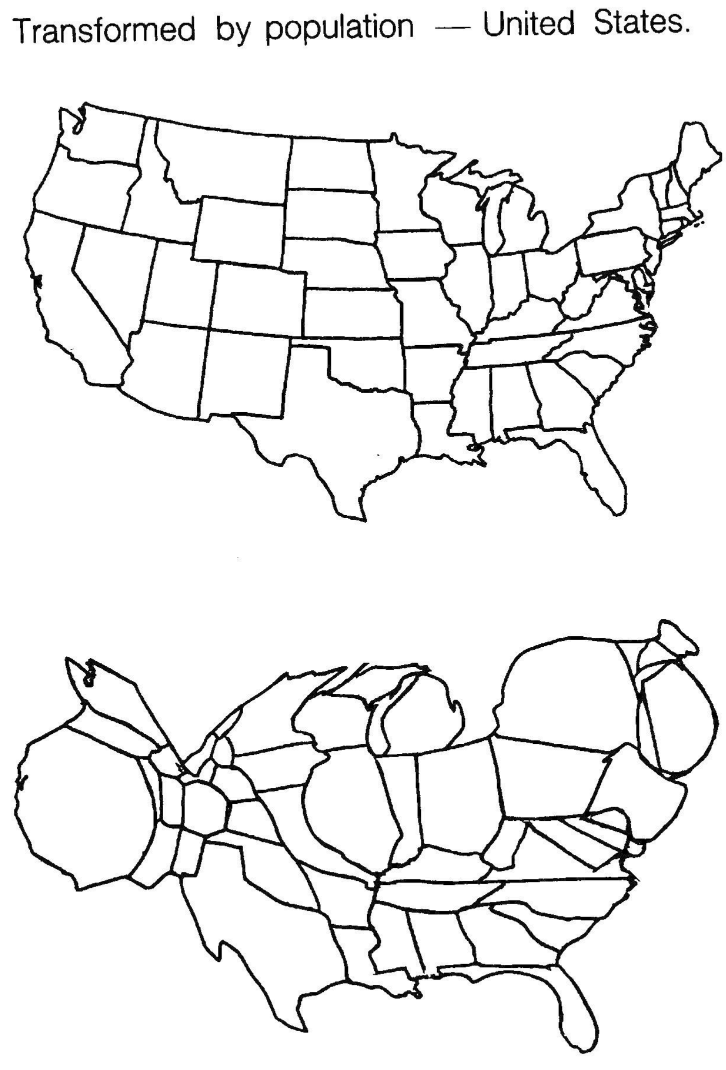


Fig. 52: Anamorphosis of the territory of the US based on the density of population.

Fig. 53: The initial image of San Francisco.

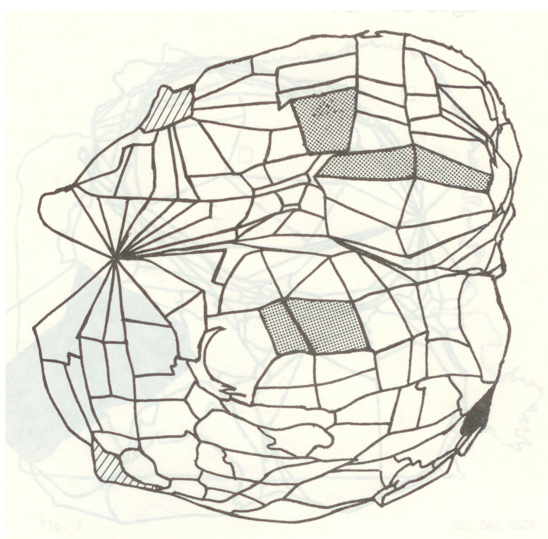


Fig. 54: The anamorphosis of San Francisco, based on the density of distribution of white male population (35–54 years old).

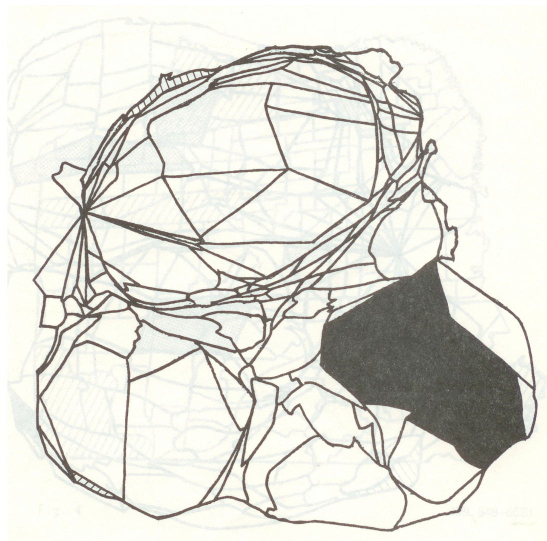


Fig. 55: The anamorphosis of San Francisco, based on the density of distribution of black male population (35–54 years old).

them which are farther from the indicated part. Therefore it is possible to expect that it is reasonable to define the influence of a cell on a vertex so that it will diminish with the increase of the distance in a suitable way. Such an approach has been used in the discussed paper. As cells there are used any connected regions , generally speaking, of any shape. If the distance from the center of a cell to a vertex is greater than $\sqrt{s_i/\pi}$, where s_i is the area of the cell, then the influence on the cell on the vertex is assumed to be inversely proportional to the distance. The length (as of a vector) of the displacement of a vertex under the influence of a cell, the distance from the center of which is equal to r, is supposed to be equal to $|R_i - \widetilde{R}_i| \cdot R_i/r$ where $R_i = \sqrt{s_i/\pi}$, $\widetilde{R}_i = \sqrt{\widetilde{s}_i/\pi}$, $\widetilde{s}_i$ is the area which the cell must have in accordance with the density (i.e. $\widetilde{s}_i = s_i \cdot p_i/\bar{p}$, where p_i is the density of the index in the cell i, $\bar{p}$ is the average density of the index on the territory). If the distance from the center of the cell is less than $\sqrt{s_i/\pi}$, then the dependence of the influence on the distance is chosen in such a way, that it is differentiable for all $r > 0$. Actually this

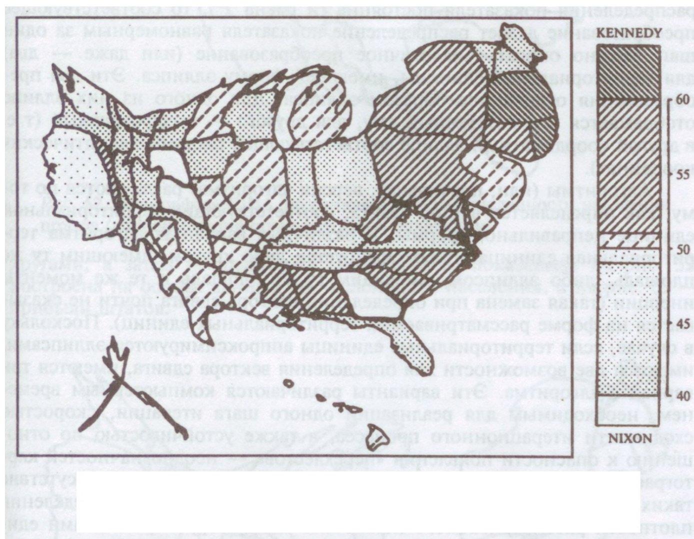


Fig. 56: Anamorphosis of the territory of US compiled with the help of the algorithm (Dougenik, Chrisman and Niemeyer, 1985)

requirement is not necessary for algorithms of such a sort, what can be easily seen when analyzing the model situation indicated above. A result of anamorphating with the use of this algorithm is shown on Fig. 56.

The essence of algorithms elaborated by the authors first (Gusein-Zade, Tikunov, 1990) is close to the algorithm of Dougenik et al. too. Therefore they demand only brief comments. The principle of compilation of an anamorphosis is based on iterative algorithms. On each iterative step for each point $\bar{r} = (x, y)$ of the territory (in the realization of the algorithm, for points from a discrete approximation of boundaries of a (usually administrative) division of the territory) there is determined the vector of displacement $\bar{d}$, equal to the sum $\sum \bar{d}_i$ of the vectors d_i of displacement corresponding to all the units of the territorial division. If the unit of the territorial division is the circle of radius R with center in the origin, P is the density of the considered index within this circle (which is, generally speaking, different from the average density $\bar{P}$ over the whole territory) then the corresponding vector of displacement d_i is equal to

$$\begin{array}{ccc} \vec{r} \cdot \left((\widetilde{R}/R) - 1\right) & \text{for} & \|\vec{r}\| = \sqrt{x^2 + y^2} \leq R, \\ \vec{r} \cdot \left(\sqrt{1 + (\widetilde{R}^2 - R^2)/r^2} - 1\right) & \text{for} & \|\vec{r}\| \geq R, \end{array}$$

where $\widetilde{R} = R \cdot \sqrt{P/\bar{P}}$ is the radius of a circle with the same total value of the index under consideration, distributed with the density, equal to the average one ($\bar{P}$). If on the rest part of the (infinite) plane the density of distribution of the index is constant (and is equal to $\bar{P}$), then the corresponding transformation makes the density uniform in one step. It is possible to describe analogous transformation (or even two transformations) for a territorial unit, which coincides with the ellipse. These two transformations differ by the image of the ellipse. For one of them it is mapped onto a radially related ellipse, for the second - onto a confocal one (i.e. onto a coordinate ellipse in the corresponding system of elliptic coordinates).

The algorithms (or rather versions of the algorithm) differ by definition of the vector of displacement corresponding to a territorial unit with an irregular shape. In versions of the algorithm a territorial unit is approximated by the circle with the center in the center of mass of the unit and with the same area or by the ellipse with the same area and the same moments of inertia. Since in the case, when territorial units are approximated by ellipses, we have two possibilities for the definition of the vector of displacement, we have three versions of the algorithm. These versions differ by the computer time necessary for realization of one iterative step, by the speed of convergence of the iterative process and by the stability with respect to the possibility of appearance of "overlappings", i.e. of non-uniquenesses of the graphical image. In order to guarantee absence of such "overlappings" in the presence of large differences between densities of distribution of the index for territorial units, it is possible to multiply the vector of displacement by a corrective factor chosen between 0 and 1. It slows down the convergence of the algorithm considerably. In any case if territorial units have complicated shapes, then the convergence of algorithms is slow. The same situation takes place if there are large

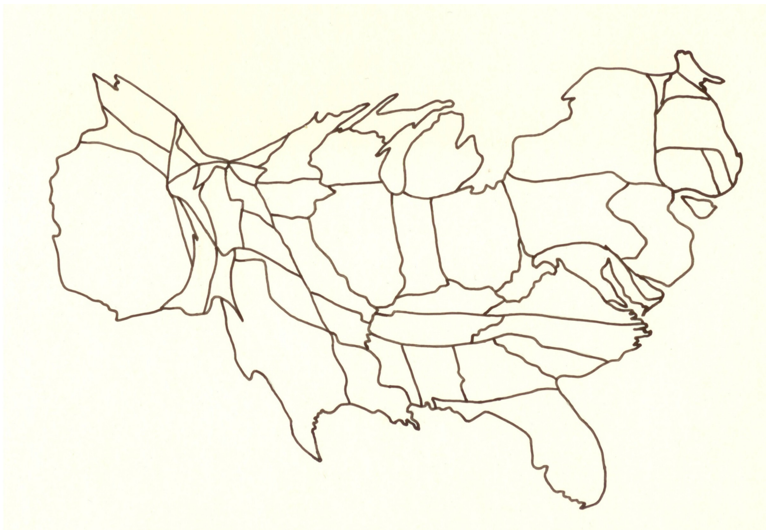


Fig. 57: Anamorphosis of the territory of US based on the numbers of population of states (approximation of states by circles and ellipces).

differences between densities of distribution of the index for adjacent territorial units. In this case after several iterative steps territorial units acquire complicated shapes after several iterative steps.

Sometimes (for example, for compilation of anamorphosis of the territory of USA in Fig. 57 and 58) there was used a combination of these versions, in which territorial units were approximated by circles in the beginning (i.e. for several first iterative steps) and by ellipses after that. The anamorphosis shown on the Fig. 57 has been compiled on the base of number of population, on the Fig. 58 - of revenues of states.

An anamorphosis of the territory of UK based based on the population density was compiled in (Dorling, 1993, a, b). For its compilation (small) territorial units were substituted by circles with areas proportional to the population. These circles (with the help of a computer program) were situated on the plane with preservation of the nighbourhood relation. The initial image with the territorial division and the anamorphated image are shown on Fig. 59. The anamorphosis

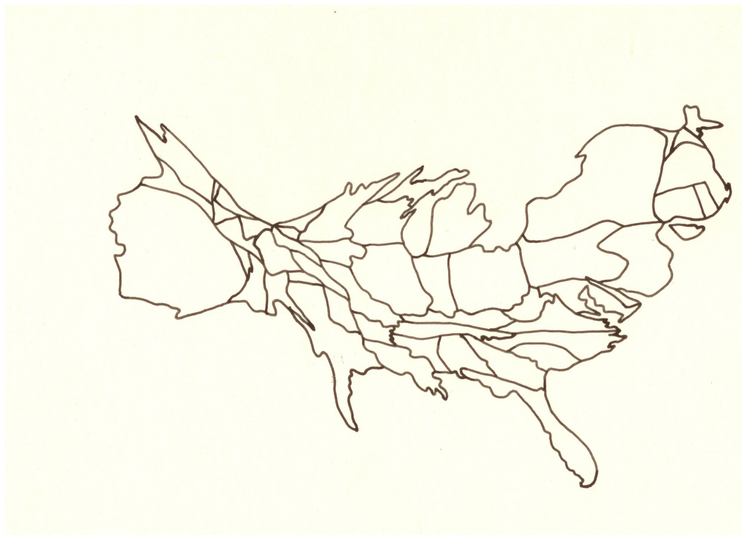


Fig. 58: Anamorphosis of the territory of US based on the revenues of states (approximation of states by circles and ellipces).

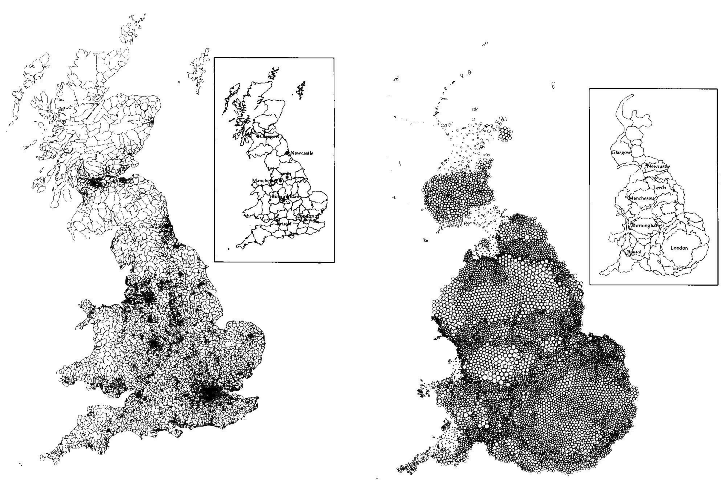


Fig. 59: Anamorphosis of Great Britain compiled with the help of the method of D.Dorling (1993).

was used for visualization of the distribution of a number of social-economic indices.

III.7. The author's method of compilation of anamorphated images.

Assume that on a region $\mathfrak{D}$ (the cartographic image of the territory) there is defined a positive density function $p(z)$ ($z = (x, y)$ is a point of the plane). Let us define the function $p(z)$ on the whole plane defining it to be equal to a constant p (say, to it average value $\bar{p}$ in the region $\mathfrak{D}$) outside $\mathfrak{D}$. Our aim is to construct a transformation $h : \mathbb{R}^2 \to \mathbb{R}^2$, which makes the density $p(z)$ uniform, i.e., such that $\det(dh)(z) = p(z)/\bar{p}$ (in particular, outside $\mathfrak{D}$, where $p(z) = \bar{p}$, the transformation h should preserve the area). Here dh is the differential of the transformation h: if h is determined by the formulae $u = u(x, y)$, $v = v(x, y)$, then $det(dh) = \frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y} - \frac{\partial U}{\partial y} \cdot \frac{\partial V}{\partial x}$. Moreover the construction should be as invariant as possible (in particular, with respect to the choice of a Cartezian coordinate system on the plane $\mathbb{R}^2$).

One should note that for a practical construction of an anamorphosis the distribution of the considered index over the territory usually is determined by its values for units of a territorial division. For example, the distribution of the population over the territory of the USA can be determined by populations of states. The density of its distribution $p(z)$ is assumed to be constant within the considered territorial units. In this case, for the extension of the density function $p(z)$ to the whole plane $\mathbb{R}^2$, it is not necessary to set it to be equal to one and the same constant $\bar{p}$ outside the considered territory $\mathfrak{D}$. Such definition can be inconvenient for the components of the complement, which lie outside the considered territory (e.g., for lakes, interior seas, inland territories which do not belong to the considered country, ...). In this case, on such a component of the complement the function $p(z)$ can be set to be equal to another constant, e.g., to the average density of population of the territorial units adjacent to it. In order not to specify this situation specially, let us assume that all components, on which $p \neq \bar{p}$, are included into $\mathfrak{D}$.

The principal idea of the algorithm. The idea of construction of the required transformation of anamorphating h can be described in the following way. We shall try to construct a one-parameter family of transformations h_t ($t \in [0,1]$) of the plane which begins with the identity transformation $h_0 = id$ and ends with the required transformation $h_1 = h$. Moreover the family h must be a solution of the differential equation

$$\frac{d}{dt} h_t(z) = \vec{\psi}_t(z)\,, \tag{19}$$

where $\vec{\psi}_t(z) = (\alpha(z), \beta(z))$ is a vector field which is determined by the function $p_t(z)$ of the density of the regarded index after the application of the transformation h_t.

First of all let us describe the construction of the vector field $\vec{\psi}_0(z)$ (which is determined by the initial distribution of the density $p(z)$). For it let us divide the considered region $\mathfrak{D}$ into a large number of very small cells. Let us regard one of these cells with the area Δs. The value of the density function $p(z)$ on this cell can be assumed constant and equal to $p' = p(z')$ where $z' = (x', y')$ is a point of the cell. For such cell let us determine an infinitesimal (i.e. infinitely small) transformation which leads the density, equal to p' inside the cell and to p outside it, to the constant p. It transforms the regarded cell into a cell with the area $\Delta \tilde{s} = p' \cdot \Delta s / \bar{p}$. Such transformation is determined by the displacement on a (also infinitesimal) vector field. Let us define the value $\vec{\psi}_0(z_0)$ of the vector field $\vec{\psi}_0$ at a point $z_0 = (x_0, y_0)$ as the sum of vectors of displacement of the point z_0, corresponding to all the regarded cells. In the limit (as sizes of all cells turn to zero) the vector $\vec{\psi}_0(z_0)$ as the sum of infinitely big number of infinitely small summands will be expressed in the form of an integral.

For a description of the (infinitesimal) transformation, which makes uniform the density equal to p' in a (infinitely small) cell and to p outside it, without loss of generality we can assume that this cell is a circle (with area Δs) centered at the point z'. Let $R = \sqrt{\Delta S/\pi}$ be the radius of this circle. The natural transformation which has the desired properties transforms this circle into the circle with the same centre and with the radius $\widetilde{R} = \sqrt{\Delta \widetilde{S}/\pi}$ by a dilatation and preserves the area (and consequently the density of the index under consideration) on the remaining part of the plane. Let us choose the polar system of

coordinates with the origin in the point z' (the centre of the cell). It can be easily seen that in such coordinates the required transformation can be defined by $(r, \varphi) \to (\widetilde{r}, \widetilde{\varphi})$, where $\widetilde{\varphi} = \varphi$,

$$\begin{aligned} \widetilde{r} &= \left(\widetilde{R}/R\right) r \quad \text{for } r \leq R, \\ \widetilde{r} &= \sqrt{r^2 + (\widetilde{R}^2 - R^2)} \quad \text{for } r \geq R, \end{aligned} \tag{20}$$

Let $\vec{r}$ be the radius vector of the point $z_0 = (x_0, y_0)$ with the beginning in the center z' of the cell (i.e. $\vec{r} = (x_0 - x', y_0 - y')$), let $r = \|\vec{r}\|$ be its length. It can be easily seen that if $z_0 \neq z'$ and the area ΔS of the cell is small enough, then the vector of translation of the point z_0 is equal to $\vec{r}\left(\sqrt{1 + (\widetilde{R}^2 - R^2)/r^2} - 1\right)$ (if $z_0 = z'$ then the corresponding vector of translation is equal to zero). We have

$$\begin{aligned} \sqrt{1 + (\widetilde{R}^2 - R^2)/r^2} - 1\sqrt{1 + (\Delta\widetilde{S} - \Delta S)/\pi r^2} - 1 = \\ = \sqrt{1 + ((p/\bar{p}) - 1)\,\Delta S/\pi r^2} - 1\,((p/\bar{p}) - 1)\,\Delta S/2\pi r^2 + o(\Delta S)\,. \end{aligned} \tag{21}$$

The last equation is a consequence of the formula $\sqrt{1+\varepsilon}1 + \frac{\varepsilon}{2} + o(\varepsilon)$ (where, as it is customary in calculus, $o(\varepsilon)$ is a value for which $o(\varepsilon)/\varepsilon \to 0$ as $\varepsilon \to 0$). Thus up to terms of the second order with respect to ΔS the sought (infinitesimal) vector field is equal to $\left(\frac{p}{\bar{p}} - 1\right) \cdot \frac{\vec{r}\cdot\Delta S}{2\pi\cdot r^2}$.

Taking sum of these vectors for all the cells of the partitioning of the territory $\mathfrak{D}$ and going to the limit as the sizes of the cells turn to zero, we shall obtain for the vector $\vec{\psi}_0(z_0)$ the expression in the form of the integral

$$\int\limits_{\mathfrak{D}} \left(\frac{p - \bar{p}}{\bar{p}}\right) \cdot \frac{\vec{r}}{2\pi \cdot r^2} dS. \tag{22}$$

Basic formulae used for the realization of the algorithm. If the density function $p(z)$ is determined as constant on each unit of a (fixed) division of the territory $\mathfrak{D}$ and is equal to p_i on the part $\mathfrak{D}_i$

($\mathfrak{D} = \bigcup_i \mathfrak{D}_i$) then the vector $\vec{\psi}_0(z_0)$ is equal to the sum over all parts $\mathfrak{D}_i$ (i.e. over all their numbers i) of the integrals

$$\frac{1}{2\pi} \cdot \left(\frac{p_i - \bar{p}}{\bar{p}}\right) \int\limits_{\mathfrak{D}_i} \frac{\vec{r}}{r^2} \cdot dS. \tag{23}$$

Using the Stokes formula

$$\int\limits_{S_i} (Pdx + Qdy) = \int\limits_{\mathfrak{D}_i} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \cdot dxdy, \tag{24}$$

One can express the integral over the part $\mathfrak{D}_i$ of the region $\mathfrak{D}$ in terms of an integral along its boundary $S_i = \partial \mathfrak{D}_i$. It is convenient to write the corresponding formula in coordinates (let us remind that $\vec{r} = (x_0 - x', y_0 - y')$). In accordance with the Stokes formula we have

$$\begin{gathered} \int\limits_{\mathfrak{D}_i} \tfrac{\vec{r}}{r^2} \cdot dS = \left(\int\limits_{\mathfrak{D}_i} \tfrac{x_0 - x'}{(x_0-x')^2 + (y_0-y')^2} \cdot dx'dy', \right. \\ \left. \int\limits_{\mathfrak{D}_i} \tfrac{y_0 - y'}{(x_0-x')^2 + (y_0-y')^2} \cdot dx'dy', \right) = \\ = \left(\left(-\tfrac{1}{2}\right) \cdot \int\limits_{S_i} \ln\left[(x_0 - x')^2 + (y_0 - y')^2\right] dy', \right. \\ \left. \tfrac{1}{2} \cdot \int\limits_{S_i} \ln\left[(x_0 - x')^2 + (y_0 - y')^2\right] dy' \right). \end{gathered} \tag{25}$$

For example for the first component we have

$$\begin{gathered} \int\limits_{S_i} \ln\left[(x_0 - x')^2 + (y_0 - y')^2\right] dy' = \\ \int\limits_{\mathfrak{D}_i} \tfrac{\partial}{\partial x'} \left(\ln\left[(x_0 - x')^2 + (y_0 - y')^2\right]\right) \cdot dx'dy' = \\ = (-2) \cdot \int\limits_{\mathfrak{D}_i} \tfrac{(x_0 - x')}{(x_0-x')^2 + (y_0-y')^2} \cdot dx'dy'. \end{gathered} \tag{26}$$

If we consider a discrete approximation of the situation, then the parts $\mathfrak{D}_i$ are polygons and their boundaries S_i are polygonal lines (with short enough edges). So the integrals along the boundaries S_i are equal to sums of integrals along some segments. The integral along the segment which connects points $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ of the expression

$$\ln\left[(x_0 - x')^2 + (y_0 - y')^2\right] \cdot dx' \qquad \ln\left[(x_0 - x')^2 + (y_0 - y')^2\right] \cdot dy' \tag{27}$$

can be reduced to the indefinite integral of the form $\int \ln x \cdot dx = x \cdot \ln x - x$ if the point $z_0 = (x_0, y_0)$ lies on the straight line passing through the points $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ and to the indefinite integral of the form $\int \ln(x^2 + a^2) \cdot dx = x \cdot \ln(x^2 + a^2) - 2x + 2a \cdot \operatorname{arctg}\,(x/a)$ if the point z_0 does not lie on this straight line. Thus the indicated integrals can be computed explicitly.

Realization of the iterative process. Let $p_t(z)$ be the density function changed after the application of the transformation h_t. The construction of the vector field $\vec{\psi}_t(z)$ from the density function $p_t(z)$ differs from the construction of the vector field $\vec{\psi}_0(z)$ from the density function $p_0(z) = p(z)$ only because the distribution of the index with the density function $p_t(z)$ has to be made uniform during the remaining period of time $(1 - t)$ and not during the period 1 as the initial distribution with the density $p_0(z) = p(z)$. So the literal repetition of the described construction leads us to a vector field $\vec{\psi}_t^*(z)$ (determined by the formula

$$\vec{\psi}_t^*(z_0) = \int\limits_{\mathfrak{D}} \left(\frac{p(z') - \bar{p}}{\bar{p}}\right) \cdot \frac{\vec{r}}{2\pi r^2} \cdot dS\,, \tag{28}$$

where $\vec{r} = \vec{z' z_0}$) which differs from the required vector field $\vec{\psi}_t(z)$ only by the factor $(1-t)$: $\vec{\psi}_t^*(z) = (1-t) \cdot \vec{\psi}_t(z)$, i.e. $\vec{\psi}_t(z) = \vec{\psi}_t^*(z)/(1-t)$. It can be proved that the solution of the differential equation $\frac{d}{dt}h_t(z) = \vec{\psi}_t(z)$ with the described right hand side $\vec{\psi}_t(z)$ (which is determined by the density function $p_t(z)$ i.e. by the transformation $h_t(z)$) indeed leads to the transformation $h(z) = h_1(z)$ which makes the density $p(z) = p_0(z)$ uniform.

In practice the differential equation (19) is solved approximately with the use of the Euler method (i.e. the method of tangents). Concretely it means the following. Choosing the interval in t equal to $1/n$

(usually we used $n = 5$), we construct the iterative process, first n steps of which are determined by the formulas

$$h_{(0)}(z) = z, \qquad h_{(i)}(z)h_{(i-1)}(z) + c_i\vec{\psi}^*_{(i-1)}(z), \tag{29}$$

where $i = 1, 2, \ldots, n$, the vector field $\vec{\psi}^*_{(i-1)}(z)$ is defined by the described above method from the density function $p_{(i-1)}(z)$ which is the result of the action of the transformation $h_{(i-1)}$ on the initial distribution of the index, $c_i = 1/(n - i + 1)$ is the constant coefficient (its form is explained by connected with the difference between $\vec{\psi}^*_t(z)$ and the right part $\vec{\psi}_t(z)$ of the equation (19)).

The transformation $h_{(n)}(z)$ is an (obtained with the use of the Euler method) approximation to the required transformation $h(z)$, which makes the initial distribution uniform. Therefore the transformation $h_{(n)}(z)$ does not make the initial density function exactly uniform. To construct a more precise approximation to the desired Transformation, the step $h_{(i)}(z)h_{(i-1)}(z) + \vec{\psi}^*_{(i-1)}(z)$ is repeated for $i = (n+1)$, $(n+2)$, ... The process finishes when the difference between the obtained density of the distribution of the regarded index and $\bar{p}$ becomes less than a chosen in advance value ε (in practice we used $\varepsilon = 0,01 \cdot \bar{p}$).

One may indicate a possibility to create animated anamorphated images. The essence of the idea (N.Bogomolov, I.Rylskiy, V.Tikunov, 2002) is the following. Assume that one has 2 images: the initial (nondeformed) one and the resulting one (an anamorphosis). Let both images (in vector forms) have the same topology and the same number of vertices for each contour. On the base of these two images a computer program creates intermediate transition phases from the initial map to the anamorphosis. A user has the option to construct all the intermediate phases of the animaion and to see them on a screen with the speed equal to 25 phases per second. The duration of an animation is determined by the user. The is the option to show the animation both from the conventional map to the anamorphosis and in the reverse direction. For construction of the intermediate phases one makes the linear interpolation of the coordinates X and Y of all the vertices of each contour. A user has the following options:
1. To look through the KRLB type file (the resulting format of the program of compilation of anamorphsises).

2. Fixing 2 KRLB files (with the same topology and the same number of vertices), to look through all the phases of transformation of one of the images into the other one. This can be a conventional map transformed into an anamorphosis and also a transition between two fixed anamorphosises (for example, between anamorphosises of the World for different years).
3. There are possible both manual change of stills and the automatic one. For the automatical change the user determines the duration of the animation and its directio (from the map 1 to the map 2 or the other way round).
4. The results of the automatical compilation of the phases may be saved as separate stills in the *.BMP format. The user has only to fix the number of the stills. Besides that, there is the possibility to choose the resulution of the obtained images (practically of any size), and also to cut and to enlarge any part of the image.
5. By compilation of anamorphosises (if the number of vertices is not sufficient) one may meet formation of several contours from one (appearance of selfintersecting lines). The user can check a KRLB file on presence of such places (if they are found, the program marks them by a contrast colour, say, by the red one).

Chapter IV. Compilation of three-dimensional anamorphosises.

It is known that the classical cartography offers a widely used series of methods of representation of images such as the colour, shading, symbolic marks etc. With the use of usual methods (for example, of column diagrams) one can show a lot of indices simultaneously (up to 30 to 40 of them), as one may meet on social-economic maps. However, clearness of such a map drops catastrophically and the quality of its perception start to approach the quality of perception of the data table on the base of which it was compiled. Just because of that, on maps, one tries to use different representational means. Besides that, the effectiveness of the used representational method is of importance. There was elaborated a method of compilation and use of a new style of representation, which, in principle, is somewhat similar to the classical method of column diagrams, but possesses a number of advantages and a higher effectiveness of representation of an index used for mapping.

The essence of the method (N.Bogomolov, I.Rylskiy, V.Tikunov, 2002) is the following. Assume that there are given 2 indices required to be mapped, for example, the real GNP and the number of population for countries of the World. In the example under consideration the indices were taken for the year 1996. (Note that the real GNP is the GNP measured in international dollars on the base of parity of purchasing power of currencies. The international dollar has the same purchasing power with respect to the GNP as the US dollar inside United States. The real GNP of developing countries is usually considerably bigger than their nominal GNP and reflects the level of the well-being of their population more adequately. In contrast to that, the potential of interaction of the real GNP (i.e., of the GNP measured on the base of the parity of purchasing power of currencies) is determined by the value of the nominal GNP, which, in developing countries, usually is lower than the real one.) In accordance with the number of population and the GNP, one compiles anamorphosises; the results are the two-dimensional images shown on Fig. 60 and 61).

Any one of them can be used as the base one. In the dicussed examples the number of population was used as the base of anamorphosises,

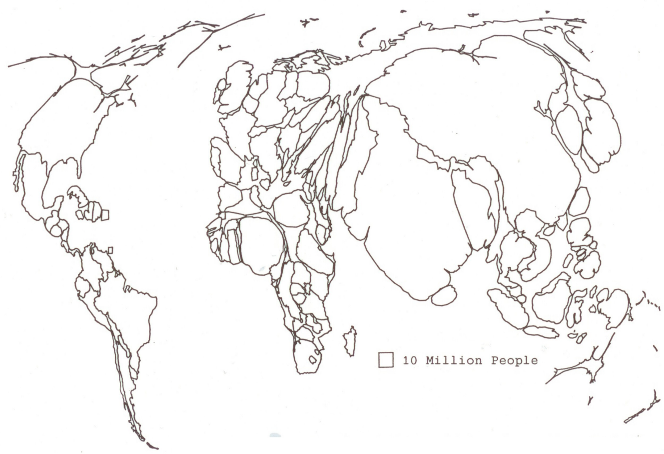


Fig. 60: Anamorphosis of the World based on the numbers of population of the countries (1996).

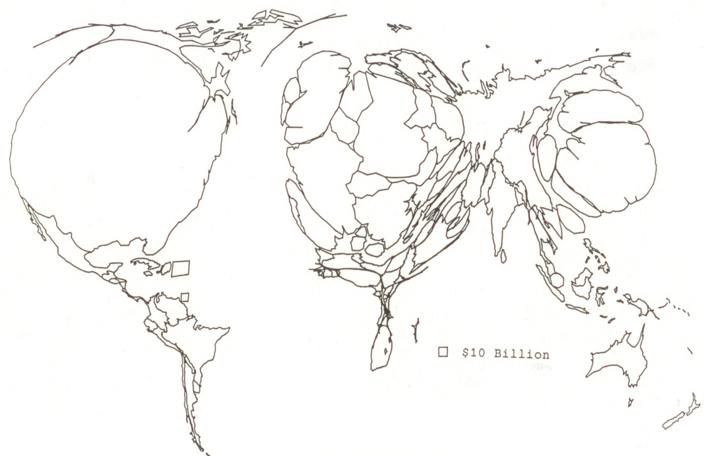


Fig. 61: Anamorphosis of the World based on the real GNP of the countries (1996).

and the GNP was shown on its background. For each country (if a counry has several disjoint parts, then for each part) one chooses a point used as its "center". The choice is made manually; the necessary condition is that the point is inside the contour of the country (if a counry has several disjoint parts, then for each part respectively). The choice of centers is made in the Cartesian (X, Y) system of coordinates on the anamorphatd image. All point of the state boundaries are assigned with the value of the Z-coordinate equal to zero and all centers of states get the value of Z equal to the value of the index under consideration (in our case - the per capita real GNP). If now using the obtained set of points one constructs the corresponding surface (automatically, using the method of Delaunay triangulation), one obtains a set of polyhedral pyramids with bases coinsiding with the country territories and with the numbers of faces equal to the number of verticies used for digitizing the boundaries of the countries.

The anamorphosis of the World based on the population possesses the following property: the area of each country on it is proportional to its population. The height of the constructed pyramid is proportional to the per capita GNP. The volume of a pyramid is equal to $V = SH/3$, where S is the area of the base, H is the height of the pyramid. Therefore the volume of the constructed pyramid is proportional to the GNP of the country.

Let us note that it is somewhat difficult to compare GNPs of two countries by sight, since the outline of each pyramid is unique and all of them have rather irregular shapes.

As a positive feature of this image, for sure, one should consider the showiness of the resulting image. It is more easy to make comparisons and to detect anomalies. One depicts two indices, in fact without using methods generally accepted in cartography, keeping them as a reserve. All the information about the two characteristics of objects are contained in the image.

It is also possible to construct column diagrams based on an anamorphosis, however the result is read and apprehended not so well since some countries cover other ones, countries with low GNP surrounded by neighbours with high GNP are simply not seen. The zero level is not well seen. Pyramid diagrams are free from this defect. They show the zero level for each contour of a country. One has a good field of vision, practically each country can be seen.

The technology of creation of such images is rather simple. The construction of the image was based on the political map of the World and the data on the number of population and real GNP for 1996. The initial map of the World was transformed into an anamorphosis using the algorithm of S.M.Gusein-Zade and V.S.Tikunov, 1999 (see section 3.7 above). The resulting data (files with the coordinates of points and with the topology of objects) were converted into the GIS ArcView 3.0 format for subsequent export into the raster format *.JPG. There were exported 2 files of equal size rasters: one — black and white (1) with contours of country boundaries and the othe one — coloured (2) with a unique colour shading for each country. The black and white raster was automatically converted into the vector format and attributed with the help of the program Raster2Vector (Able Software), there were put and attributed central points of polygons. The result was saved in a file of *.SDL format and then converted into a *.CON file. The latter one was the starting-point for the program DEMI, which constructs surfaces for a given set of irregularly arranged points. The result was saved in a file of *.DEM format, which represents a regular net of points with determined heights. With the help of a convertor the file was saved in the *.BMP format (with grades of grey: more white for higher levels and more black for lower ones) After that (on the base of the obtained BMP file) the package 3D-Studio MAX 2.5 produced, from a regular net of size 300×300, the three-dimensional surface with the pyramidal block-diagrams. After that this surface was covered by a coloured raster image: for making it easier to use the image. Since for a detailed consideration of the surface one should look on it from different sides, there were created several images of different regions of the World (Fig. 1 and 2 of the coloured inset).

As in the case of area anamorphosises, three-dimensional anamorphosises can be animated. Some general remarks first. As in the case of area anamorphosises, tree-dimensional ones also can be animated. With the start of the era of personal computers, which permit to reproduce full scale videos, one equates a possibility to show an object in dynamics to increasing the dimension of the initial static image by one level. Dynamic 2D-images (for examples cartograms where values of an index are shown by colour) are, as a matter of fact, three dimensional.

Dynamic series of data for a series of indices are often met in the social- economic cartography. Construction of dynamic 2D images for each of the indices became a rather usual event. Analysis of one cartofilm is also not complicated. It is also possible to construct a cartofilm for each of the indices. However it is somewhat difficult to find correlations between them. Thus we have arrived to the essence of one of the problems: depicting several dynamic indices simultaneously without constructing additional dynamic maps of correlations.

To consider the problem, let us use 3 indices: the number of population, per capita GNP, and average length of human life. All the indices were taken for each country of the World for last 30 years. Showing the dynamics of all three indices on one model using methods of classical cartography is rather complicated. However, a solution can be the following one.

1. The population of the World: for each year one constructs an anamorphosis of the world population. Each of the images is two dimensional. If one shows the images subsequently, one after another, we see "inflations" and "compressions" of different countries. Since the images are raster ones, for each country let us choose a point which is not displaced for all the period and is situated approximately at the center of the country and let us determine its XY- coordinates. One apprehends even small shape changes well enough and therefore the dynamic of the phenomenon can be seen very well.

2. Expected length of human life: its dynamics may be represented by colour. Let us choose a multi-level colour scale (say, 100 levels of colour) so that each level of the index is denoted by one colour). According to this, the dynamics of the length of human life in each country is reflected by a sufficiently continuous change of its colour. This is also well apprehended.

3. Per capita GNP. For that one uses the method of 3D-pyramidal block-diagrams. The essence of the method is as follows. Let one have a 2-dimensional map of countries (or of other territorial units). For each country one chooses (by sight) a center (X, Y) and the corre-

sponding value of the Z-coordinate is the per capita value of GNP for this country in a given year. All verticies of contours of countries have their own XY-coordinates and the value of Z for them let be equal to 0. If, for the obtained XYZ-mass, we construct the surface in the 3-space and, for better orientation, cover it by the initial 2-dimensional map, we shall see that each country became "a mountain", the base of which has the shape of the country and the height is proportional to the value of the index under consideration (per capita GNP in our case). The initial twodimensional map may be both the classical political map of the World and an anamorphosis, static or dynamic. A necessary condition is that XY-coordinates of the "mountain summit" for each country should not change in time.

Taking into account all discussed above, there were constructed dynamic pyramidal block-diagrams for GNP, moreover their "summits" were kept (on the image) at the same places and their heights changed in time. Shapes and areas of bases of these "mountains" changes depending on the country population (a dynamic anamorphosis). At the same time each "mountain" changes its colour in complete accordance with the expected length of human life.

To make such images metric, values of the indices can be simply written over the summits of the "mountains". It is also possible to draw the level lines. "Dimension" of this image is very high. Indeed, a static non-coloured image would be a visualization of a certain surface. In time there are changing heights of the "mountains", shapes of their bases, and their colours.

Chapter V. Examples of application of anamorphosises

V.1. Visual analysis of anamorphosises

In the history of compilation and use of anamorphosises it is possible to find examples concerning very different fields. But mostly often they are used for representing various characteristics of population on territories, in electoral and medical geography, for representing quality of the environment, air pollution and so on (Levison, Haddon, 1965; Forster, 1966; Härö, 1968; Ruston, 1971; Perspective..., 1977; Pravda, 1977; Sliwa, 1978; Wonders, 1980; Bochkareva, 1981, 1983; Kadmon, 1983; Selvin et al.,1984; Kelly, Neville, 1985; Belov, 1988; Uzan, 1989 et al.). In spite of the variety of methods used earlier for compilation of anamorphosis, in this chapter we shall use only examples compiled with the algorithm, elaborated by the authors. Fist of all let us look at the images of the world. On Fig. **??**, there is an anamorphosis compiled on the base of the numbers of population of countries (data and boundaries of 1989). Countries are easily recognizable by their shapes (configuration). Of course the most attention is attracted by China and India. In Asia only Mongolia and Laos look more than unpretentious against the background of their encirclement. It is interesting that even such little "specks" on the initial map as Hong Kong and Singapore turn into considerable territories on the anamorphosis. Australia is "shrunk" rather heavily. In Africa, which is rather modest compared with Asia and even with Europe, first of all attention is attracted by the "ball-shaped" Nigeria. The most contrasts are inherent in Europe - compare the "tiny" Benelux and the countries of Scandinavia. Respectively uniformly are populated countries of America with the exception of Canada and Greenland.

Putting on an obtained anamorphosis characteristics connected with the amount of population, for example, the provision with food products, we shall get a more adequate impression about e.a. its deficiency. It will be so because it is related not to the territory, as it is on traditional maps, but more correctly - to the population which is needy in it. Now let us look at the second anamorphosis, compiled on the base of the data on the gross national product (Fig. **??**). Contrasts here are even sharper than on the previous one. First of all are striking three world "centers of welfare" - USA, Western Europe and Japan. Some countries of Europe are so heavily deformed that it is not easy to recognize them. However whereas in Europe the word deformation means expansibility, in Africa this process goes in the opposite direction. The majority of countries of Western and Central Africa are simply merged with each other. There are distinguished only the South African Republic and some oil-extracting countries. It is interesting to look at India and China - the leaders of the previous anamorphosis. Here they look more than modest. The relation between the areas of China and Hong Kong on the anamorphosis is curious. More "worthy shape" acquires Australia. In America all other countries look unpretentious against the background of the "moneybag" - the USA. Alaska has been included into consideration as a separate territory (in the sense that its own gross product has been attributed to it). It led to its "compression" into a narrow stripe. Almost regular rectangular shape of the islands of Puerto Rico and Trinidad is explained by their small sizes on the initial map. Therefore they were digitized very roughly, only with the use of four points each. On the anamorphosis their territories become more noticeable.

The series of anamorphosises of the World can be easily continued. As examples we shall show 5 more anamorphsises: Fig. 62, 63, 64, 65, and 66. However let us look at an anamorphosis of the former USSR. On the map of the USSR in the boundaries of 1990 (Fig. 67) for units of the administrative-territorial division there is shown the average sick

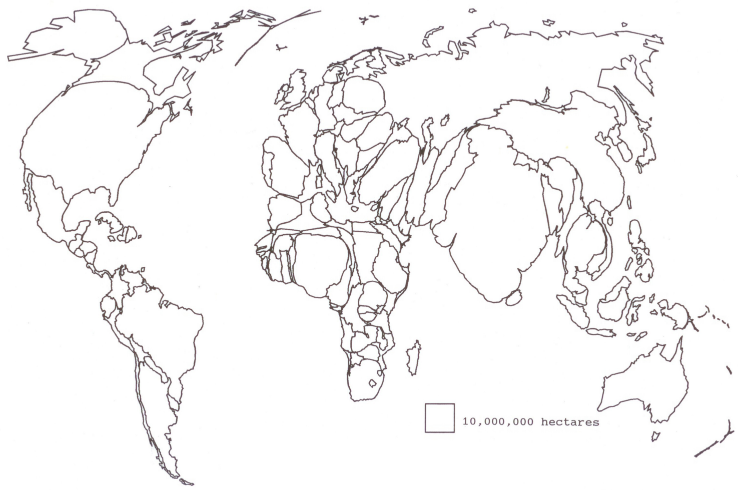


Fig. 62: Anamorphosis of the World based on the amount of arable land.

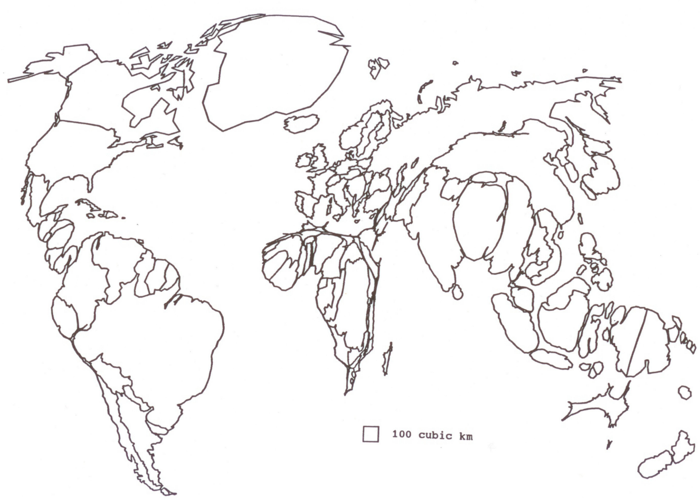


Fig. 63: Anamorphosis of the World based on the annual inner renewal water resources.

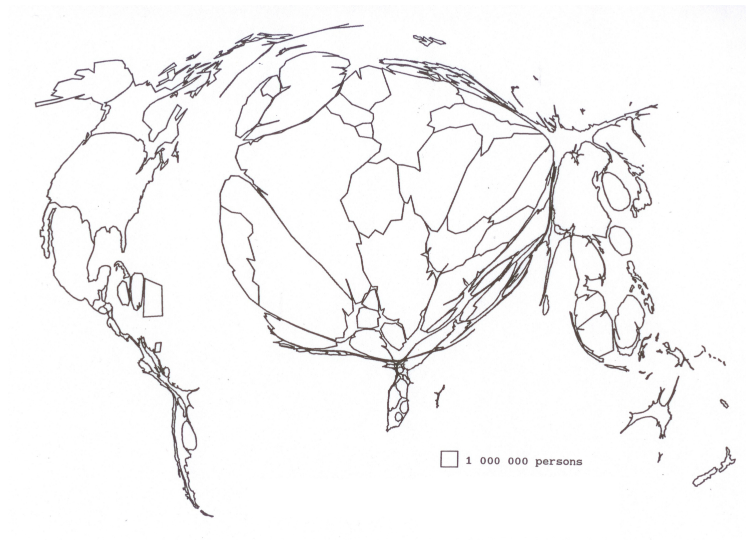
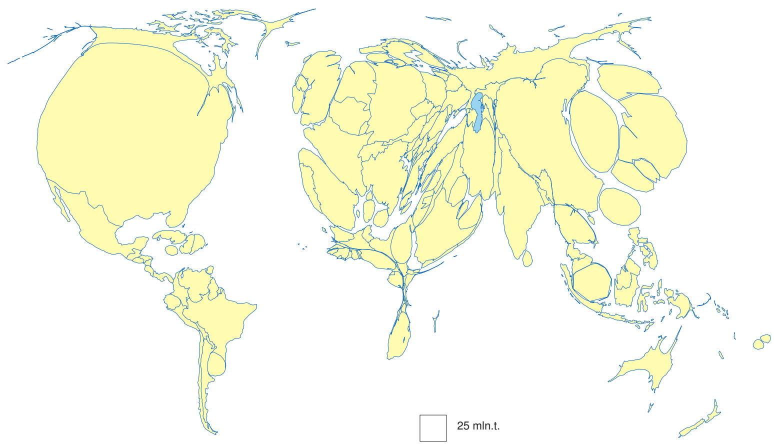


Fig. 64: Anamorphosis of the World based on the number of tourists visited the countries (1990).

rate of the infectious hepatitis in USSR in 1970-1985. On Fig. reffig66 there is an anamorphosis compiled on the base of the data on the number of population (1990). with the same sick rate shown on it. Whereas on the traditional map it seems that the level of distribution of the disease is more threatening in the Siberia in comparison with regions of Kazakhstan and the Middle Asia, on the anamorphosis the picture is quit contrary. Moreover areas with the highest sick rate are most heavily expanded. The analysis of the phenomenon on the anamorphosis does not leave doubts about priorities in rendering the medical aid and in organization of preventive measures.

Let us pay attention to the central, dominating position of the Moscow oblast (region), the most large on the anamorphosis. On the traditional map a reader can find it not at once. On the anamorphosis relations between numbers of population of the Transcaucasian republics, of the Ukraine and of the Baltic republics can be seen quit clearly.

One should note that in this case for compiling the anamorphosis



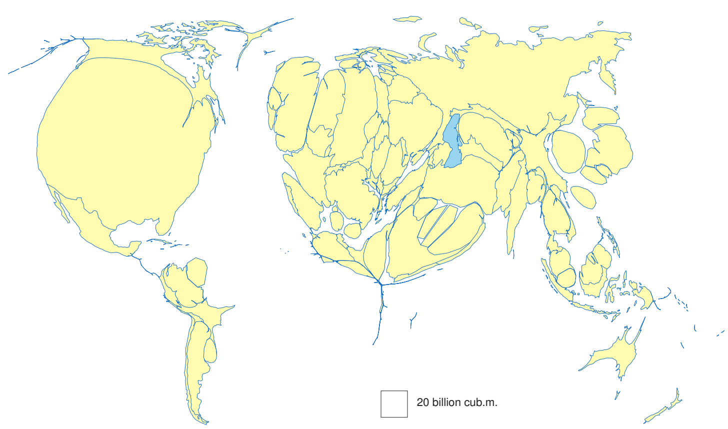
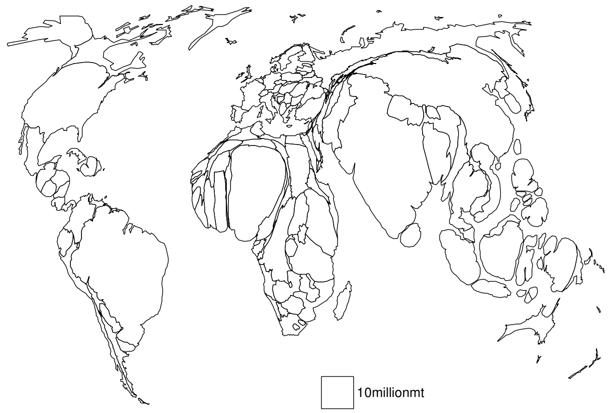


Fig. 65: A - Volume of consumption of oil in 2005. B - Volume of consumption of gas in 2005.



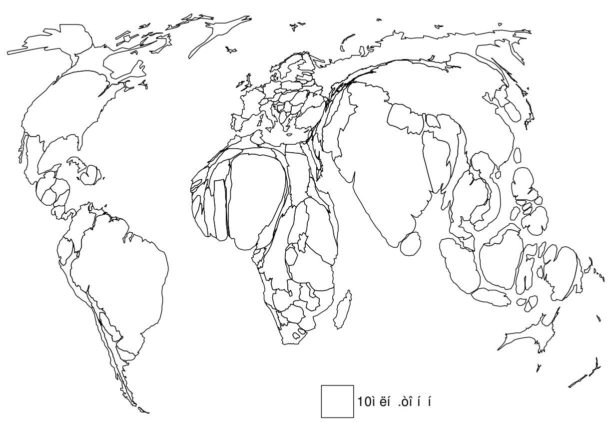


Fig. 66: A Share of carbon dioxide emissions stemming from the burning of fuelwood in total emissions. B - Emissions from the manufacture of cement, from the burning of fossil fuels and fuelwood as well as from shifts in land use.

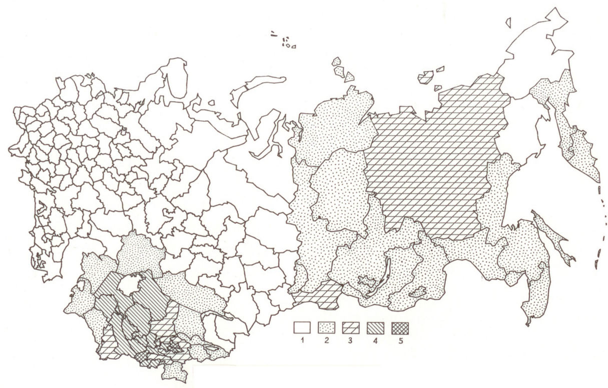


Fig. 67: Map of morbidity by the infectious hepatitis in USSR. Levels of morbidity (indices of intensivity per 100,000 persons): 1 — low (under 200); 2 — middle (200-400); 3 — heightened (400 -600); 4 — high (600-800); 5 — very high (800 and over).

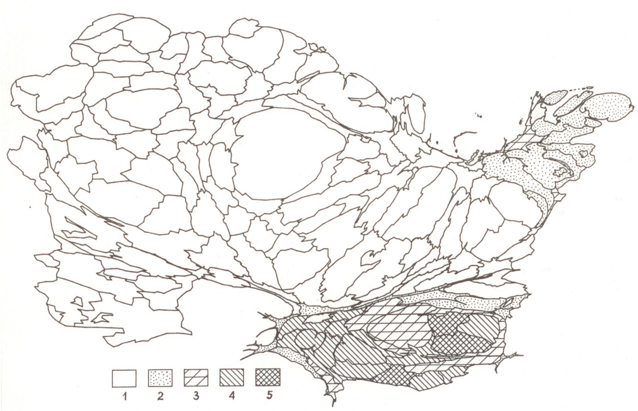


Fig. 68: Anamorphosis of USSR based on the number of population with characteristics of morbidity by the infectious hepatitis (1–5 — see Fig. 65).

of the USSR a modification of the method has been used. At first there was made a conformal transformation of the image depending on the territorial distribution of population.

This was done in order to make the distribution of population more close to the uniform one with the help of an analytically defined transformation and to quicken the convergence of the algorithm. Such transformation was defined by the formulae

$$\begin{aligned} \widetilde{x} &= C \cdot e^{ax+by} \, cos(ay - bx) \,, \\ \widetilde{y} &= C \cdot e^{ax+by} \, sin(ay - bx) \,, \end{aligned} \tag{30}$$

where a and b are the coefficients of the regression $p(x, y) = A \cdot e^{2(ax+by)}$ (more precisely, of the linear regression $\ln p(x, y) = \ln A + 2ax + 2by$; here, just as above, (x, y) are the Cartesian coordinates on the plane, $'$ is an arbitrary scale coefficient). The conformity of the transformation (30) follows from the fact that (30) is the expression in the real coordinates of the complex- analytic transformation $\widetilde{z} = C \cdot e^{(a-ib)z}$, where $z = x + iy$, $\widetilde{z} = \widetilde{x} + i\widetilde{y}$. This transformation makes uniform the density $A \cdot e^{2(ax+by)}$ because

$$\begin{gathered} |d\widetilde{z}/dz| = |C \cdot (a - ib) \cdot e^{(a-ib)z}| = \\ = |C \cdot (a - ib) \cdot e^{ax+by}| = const \cdot e^{ax+by}. \end{gathered} \tag{31}$$

Now let as draw the reader's attention to the anamorphosises of Russia using the administrative territorial division (Fig. 69) and based on the numbers of population (Fig. 70) and on the gross regional product (Fig. 71)[2].

Let us look at some meaningful indices put on anamorphosises. As an example, on the anamorphosis of Russia (Fig. 70) there is shown the territory of primary compact settlement of ethnically Russian population (Fig. 72) taken from the map of ethnic structure in the "Atlas of world nations" (Fig. 73). On the usual map the shaded area makes up 31.4increases to 78.6ethnically Russian population in Russia makes up 81,5about the ethnic structure of a country are simpler and more adequate, if they are guided not by a traditional map, but

[2] *Because of the lack of data here and further data for the Ingush and for the Chechen republics were used together.*

Fig. 69: The initial map of the administrative–territorial division of Russia. (See the numbering of administrative units in the left column of Table 2.)

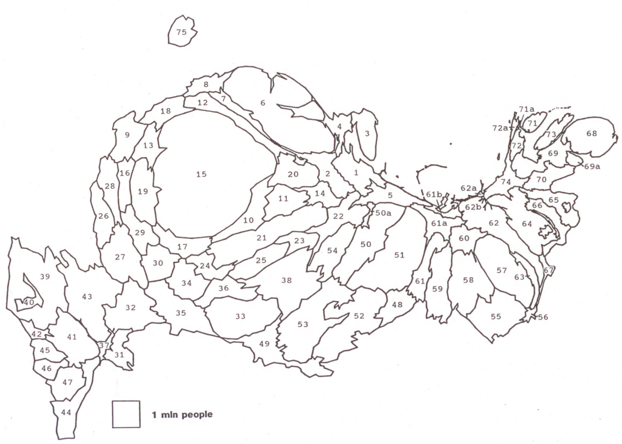


Fig. 70: The anamorphosis of Russia based on the numbers of population (2002).

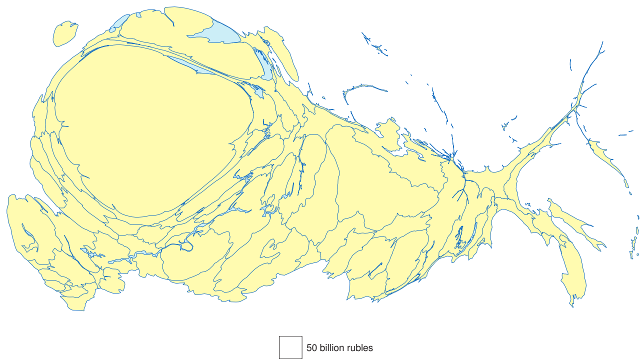


Fig. 71: The anamorphosis of Russia based on the gross regional product (2002).

Fig. 72: Anamorphosis of Russia based on the number of population with distinguished regions with prevalence of Russian population

Fig. 73: The initial map of Russia based on the number of population with distinguished regions with prevalence of Russian population.

by an anamorphotic image. Otherwise, not being acquainted with the character of the territorial distribution of the density of population (which is not shown on maps of ethnic structure), it is possible to make the absurd conclusion, that the majority of the population of Russia is formed by nations (ethnics) of the North - Nenetzes, Evenks, Chukchi, Evens, Nanaians and so on, living on huge (enormous) territories of the country. Linking this conclusion with the information about the high birth-rate, typical for nations (ethnics) of the North, also corresponded visually not to the population, but to the territory, it is possible to obtain the same inept result for characterizing the country in the whole based on data for only 0.11nations (ethnics) of the North in the total population of the Russian Federation according to the last census (Tikunov, 1986). This example is trivial, but for small and unfamiliar territories such inadequate perceptions and opinions are probable.

It is possible to give a number of examples of preferable use of anamorphosises. Thus we compiled anamorphosises of the USA based on the data on the numbers of population (Fig. 74) and on revenues of states (Fig. 75). The obtained images were used for showing different

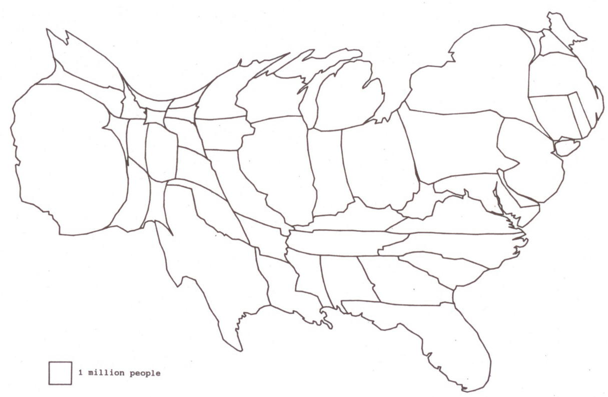


Fig. 74: Anamorphosis of the territory of USA compiled on the base of numbers of population of states.

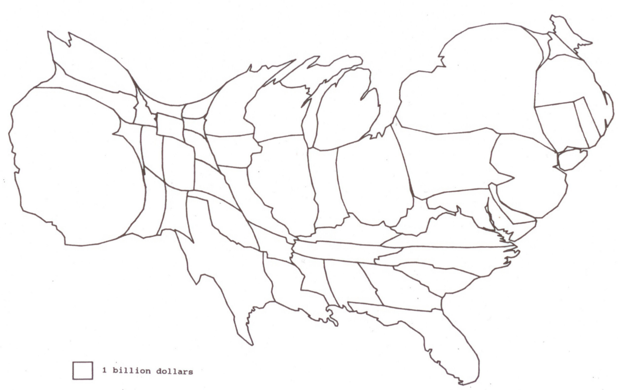


Fig. 75: Anamorphosis of the territory of USA compiled on the base of revenues of states.

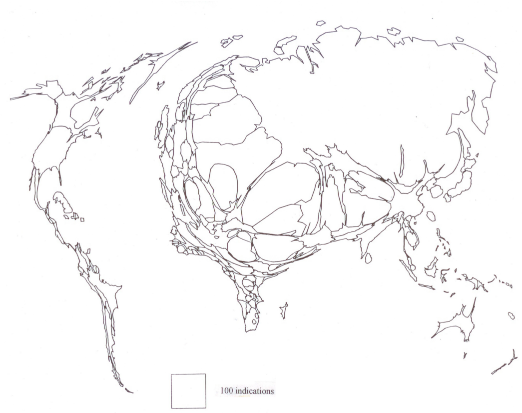


Fig. 76: An anamorphosis based on the frequency of mentioning of country's names in the "Nezavisimaya gazeta" newpaper (April 1997-1998).

indices: ethnic structure of the population, distribution of religions, rates of economic development of states, numbers of votes for different candidates at the presidential elections. They can be used for a number of other statistical indices. Anamorphosises (especially based on the numbers of population) give a variety of material for the meaningful analysis.

The series of anamorphosises of Russia can be easily continued (see, e.g., addendum to the book [Tikunov, Tsapuk, 1999] were there are examples based on the number of newborn, the number of passed away, the volume of the industrial production, investments in the basic capital, volume of retail trade for administrative-territorial units for 1996 etc.] and one can show possibilities of their practical use. Let us indicate also mapping of the frequency of mentioning of countries and Russian regions in the periodical press [Kolosov, Tikunov, Zayats, 2001], Fig. 76, and of the frequency of their mentioning in Internet in using search engines: american Altavista (www.altavista.com) and Russian Rumbler (www.rambler.ru), Fig. 77, and Yandex (www.yandex.ru) [Oreshkina, Tikunov, 2001].

Fig. 77: An anamorphosis based on the frequency of mentioning of country's names in Internet (Rambler system).

In a number of cases there arises a need in compiling an anamorphosis based on data which are not tied to any administrative or natural boundaries. For example such a situation is characteristic of maps of seas and oceans, were the information is usually tied to trapeziums of the net, formed by parallels and meridians, or to cells of other regular nets. For handling of regular nets there were worked out modifications of algorithms of compiling of anamorphosises. These modifications were used for transforming the map of the Atlantic Ocean.

The anamorphosis on the Fig. 78 has been constructed from the net formed by parallels and meridians with the step equal to 5° . It was based on the index of distribution ships in the Ocean. The initial map of density of distribution of ships in the Atlantic Ocean has been compiled on the base of the scheme "Prognosticated distribution of ships with the tonnage greater than 100 register tons in the World Ocean (including fishing-boats) for the year 1980 (Economic..., 1979). Fig. 78 shows only the coastal line and the net of parallels and meridians, but this image can be use for representing other indices, connected with the distribution of ships. Thus Fig. 79 shows the pollution of the Ocean by the oil layer in percents of (cases of) detecting. The initial

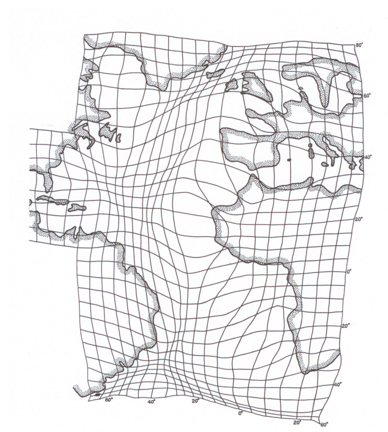


Fig. 78: The anamorphosis of the Atlantic Ocean based on the distribution of ships.

Fig. 79: The pollution of the Ocean, shown on the anamorphosis based on the distribution of ships.

map of the pollution has been taken from (Sigolaeva, Tikunov, 1986).

On the anamorphosis (Fig. 79) the areas with high level of pollution are usually increased, what testifies for the connection between the number of ships and the level of pollution. However it is possible to find other examples, as decrease of the area in the region of the Sargasso Sea where the pollution is high, but the density of ships low. It can be explained by the drift of the oil layer from other regions by several sea currents converging in this place. In the whole the oil layer meats more frequently near the western coasts of Africa, in the Mediterranean and the Caribbean Seas. Complicated distribution of the oil layer can be observed in the Gulf of Mexico and in the Caribbean Sea. Typical (inherent) for these regions spotted character of the distribution of the layer is originated under the influence of strong winds and surface currents. High average velocity of the wind can be observed to the north of the Gulf Stream and here the number of cases of detecting of the layer is small. The oil layer can be often met near the Straits of Gibraltar and in the Sargasso Sea (20-30% of all cases of detecting). Considerable amount of the oil layer has been found at the north-western coasts of Africa. From there the oil drifts to the south under the influence of the Canary and the Northern trade-wind currents and then to the West to the coasts of the Latin America, where, as it is considered, occurs its quick destruction. The analysis of regularities in distribution of the oil layer is interesting if guided by the image, based on the distribution of ships - the main polluter of the Ocean.

As another example we can indicate the anamorphosis based on the integral accesebility of aquatories for the population of the coastal zone (Fig. 80). The method of calculation of this indicator and the initial map itself can be found in the paper (Sigolaeva, Tikunov, 1986). The indicator of accessibility is determined by the well known in human geograpy formula of the induced potential of the settlement field:

$$V_j = \sum_{i=1}^{n} \frac{P_j}{D_{ij}}, \tag{32}$$

where the potential V_j in a cell j, bounded by parallels and meridians, is induced by all regarded sea ports (i) with the number of population

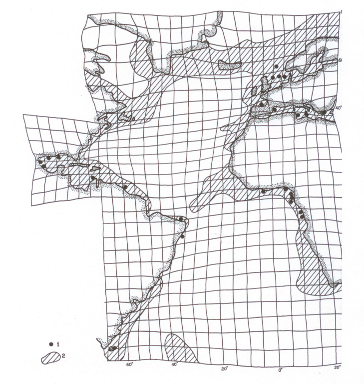


Fig. 80: The anamorphosis of the Atlantic Ocean based on an index of integral accesebility: 1) biggest centres of oil output; 2) regions of intensive fishing.

P_i greater than 10000;

$$D_{ij} = 6378,245\, arc\, cos(sin\varphi_1 \cdot sin\varphi_2 + cos\varphi_1 \cdot cos\varphi_2 \cdot cos\Delta\lambda)$$

Is the distance between the points i and j (orthodromy) calculated from geographical coordinates φ and λ. For calculations all ports of the Atlantic sea coast were taken into consideration: there are 85 such ports in Europe, 23 in Africa, 30 in South and Central America, and 36 - in North America.

As the base of the anamorphosis on Fig. 80 the index inverse to the value V_j was used. In this case the larger a cell (of the aquatory) is the less convenient in average it is for development by people and vice versa. On Fig. 80 there are shown regions of intensive fishing. It is easy to estimate visually the convenience of their arrangement for developing. In a similar way an estimation of arrangement of largest oil-gas deposits can be done.

As it is shown on the Fig. 80, biggest centres of oil output are situated in Mexican, Venezuelian, North–European and Guianian megabasins.

The shelf of the Gulf of Mexico is the most studied and intensively exploited. There are drilled more than 16,000 boring wells. There are found more tham 320 sea deposits. Total prospected reserves on the North shelf of the Gulf are estimatied to be approximately 2 billion tons of conventional fuel. It is expected that till the end of the century there will be extracted 80 million tons of oil and 100 billion m^3 of gas.

On the second place in the world (after the Persian Gulf) by the volume of oil and gas extraction there is the Maracaibo basin (a part of the Venezuelian megabasin) situated on the South shelf of the Caribbean Sea in the lagoon of Maracaibo (4,612 million tons of oil, 1,142 billion m^3 of gas). The total previewed reserves of oil on the continental shelf of the Carribean Sea are estimated to be 12.7 billon tons. The exploitation of the Guianian–Brazilian littoral started just recently.

On the shelf of Canada and the USA there are distinguished oil-gas-bearing basins Baltimore–canyon, Newfoundland shoal and Labrador. According the Fig. 80 these basins are easily developped, since they are situated close to densily populated economic centres of these countries, what increases their value. Potential reserves of oil on the Atlantic coast of the North Africa are estimated to be 7–8 billion tons. On the African coast of the Atlantic Ocean oil and gas deposits are situated in the Gulf of Guinea, on the continental shelf of Nigeria, Gabon, Angola and other countries. On the submarine outskirts of the North–Western Europe there is situated the biggest North Sea oil-gas-bearing basin (3,700 million tons of oil, 3,100 billion m^3 of gas), and also found in 70th deposits in the Ireland Sea. In the North Sea there are exploited nearly 80 oil deposits and more than 50 gas ones.

V.2. Formation of the subject structure of anamorphosises by methods of mathematical-cartographic modeling.

Mathematical-cartographic modeling (MCM) was formed from a large number of separate experiments on use of mathematical methods in the subject cartography in the beginning of 70th (V.T.Zhukov, S.N.Serbenyuk, V.S.Tikunov, 1973, 1980). Under the term "mathematic-cartographic modeling" one has in mind a fundamental composition of mathematical and cartographical models in the system "compilation-use of maps" for construction and analysis of subject content of maps. Mathematic-cartographic models may be elementary, expressed in the following way: initial data + mathematical model = a result of modeling. Under the term "data" one may understand information read from a map, "a result of modeling" might be a subject content of a map. In other words, either on the starting stage of the modeling, or on the final one, or on both two stages simultaneously there must participate a cartographic model, otherwise one cannot call such modeling mathematic-cartographic.

First of all some words about components of the mathematic-cartographic modeling: *cartographical and mathematical models.* As to a map, it presents itself as a mathematically strictly determined formal model, whose construction is made by canons of mathematical cartography.

On a map, the simulated reality, as in a mathematical model as well, is transmitted in a symbolic form. However, a map possesses a property distinguishing it from a mathematical or any other model: it visualizes a territorial reality. Just this property causes graphic clearness of cartographic characteristics of a territory and explains the centuries-old tradition and diversity of directions of use of maps in science and in practice. A map is not only an abstract symbolic, but also an analogue model of the reality. A proof of that is a variety of methods of transmission of characteristics of phenomena by means of interchangeable modes of cartographic images, and also unequivocal understanding of characteristics of specific territorial properties of the geographic reality.

In spite of differences between mathematical and cartographical models, just mathematics served as an important reason of origin and development of such methods of mapping as a cartogram or a cartodiagram, the punctual one and the method of isolines. The methods of mathematical statistis also are not exceptional; they are used for a long time in the map compilation practice by conducting a choice of mapped objects, by construction of scales for quantity indicators, by generalization of statistical data etc. New for cartography was the engrossed process of adoption of mathematical methods into formation of subject and content of maps, leading to a more deep reconstruction of methods of their compilation (V.T.Zhukov, S.N.Serbenyuk, V.S.Tikunov, 1980). All this permits to speak about a possibility of fundamental combining of mathematical and cartographical models and about pointlessness of counterposing them.

Combination of mathematical and cartographical models may be very diverse and may be expressed both in simple forms and as a complicated multistage process. The latter is as if constructed from these models-sections, which may be classified (V.S.Tikunov, 1979).

A mathematic-cartographic model as if synthesizes mathematical and cartographical elements together. In connection with this, it is not possible to classify elementary mathematic-cartographic models by types of used maps or by the used mathematical machinery. Such classification is very tempting since both cartography and mathematics already have their own divisions and corresponding classifications.

In our case, neither cartographical, no mathematical components separately Do not determine the fse of MCM. Graphically saying, mathematical machinery is like a mincing-machine which only minces, processes the data and represents them in a form more convenient for an analysis, reveals hidden regularities etc., most often fixed on maps. Based on these principles, there was elaborated a classification of *elementary mathematic-cartographic models*:

A. Models of phenomena structure.

I. Models of structure of spatial characteristics of phenomena.

II. Models of structure of subject characteristics of phenomena.

B. Models of phenomena interconnections. I. Models of interconnections of spatial characteristics of phenomena.

II. Models of interconnections of subject characteristics of phenomena.

C. Models of dynamics of distribution (development) of phenomena.

I. Models of dynamics of spatial distribution of phenomena.

II. Models of dynamics of subject development of phenomena.

Examples of construction of elementary models corresponding to all the points of the classification can be found in [V.S.Tikunov, 1985; 1997]. Here we shall show only one example for each group of models, not disclosing all the variety of them.

Thus, as an example of a realization of a model of structure of spatial characteristics of phenomena let us indicate an analysis of uniformity of distribution of the net of food shops in Troy, New-York. The corresponding anamorphosis was shown earlier: Fig. 48. As might be expected, the nature of location of food stores in an equidemic projection is more regular than on the actual earth's surface. However, even here it is possible to find deviations which must be analyzed in the development of the network of stores, if the objective is convenience of service to the entire population.

Tipical models of structure of meaningful characteristics of phenomena are classifications. As an example of such classifications one can indicate estimations of the rate of favourability of the state of the environment and of the population health and also typology of causes of morbidity and mortality of children and of the whole population by administrative units of Russia in 1991 (Malkhazova, Tikunov, 1993, b). As initial territorial units 75 regions, territories and republics have been used according to 1991 administrative division. It was this very level which allows us to take advantage of various normalized data of the State Statistical Survey for the whole Russian Federation.

Integral assessment of the state of environment was based on the values of nine parameters, describing the main factors of possible health effects: share of trapped and treated harmful substances in the total amount of emissions from stationary sources, per cent (1); share of normatively purified wastewaters in their total volume, per cent (2); degree of water pollution (number of tap water samples which do not meet sanitary standards, per 100 samples studied) according to chemical (3) and microbiological (4) parameters; level of air pollution (number of air samples which exceed maximum permissible levels, per 100 samples studied) (5); sanitary state of agricultural products and foodstuffs (number of product and foodstuff samples which do not meet sanitary standards, per 100 samples studied) according to chemical (6) and microbiological (7) parameters, nitrate (8) and pesticide (9) concentrations.

The population health was characterized by a set of 7 parameters: total number of illnesses per 1000 inhabitants (1); number of malignant tumors per 100000 inhabitants (2); total number of originally registrated child illnesses per 100000 children (3); number of malignant tumors per 100000 children (4); total mortality per 1000 inhabitants (5); infant mortality (number of children who died before they reach 1 year, per 1000 born) (6); number of births per 1000 inhabitants (7).

Thus the matrices of initial data were formed which are 75x9 and 75x7 respectively in size. The matrix data were analysed using the elaborated algorithm (Tikunov, 1985; 1997, a). At first, for the sake of comparability all initial parameters were normalized :

$$\hat{x}_{ij} = \frac{|x_{ij} - \overset{\circ}{x}_j|}{|_{max/min}x_j - \overset{\circ}{x}_j|} \tag{33}$$

$$i = 1, 2, 3, \ldots, n, \qquad j = 1, 2, 3, \ldots, m,$$

where $\hat{x}$ are the normalized initial values (x); $\overset{\circ}{x}$ is the worst value of each parameter as to the favourability of environmental state and population health; $_{max/min}x$ are the values mostly different from $\overset{\circ}{x}$; n is the number of analysed territorial units (75); m is the number of parameters (9 and 7 respectively).

As the worst values $(\overset{\circ}{x})$ for the state of environment the minimum values of the first and the second parameters and the maximum values of other parameters have been chosen; the population health was described using the maximum values of six parameters and the minimum value of the last one as the worst values.

Confronting the parameters of all territorial units with an imaginary one, characterized by the worst values of $(\overset{\circ}{x})$, these units were arranged as to both favourability of environment for population health and real state of health.

The arrangement was made using the Euclidean distances to measure the proximity of each analysed unit to an imaginary one which had the worst values of $(\overset{\circ}{x})$ for all indices 33. This required the processing of the data array using the method of the main components in order to orthogonalize and convolute the system of parameters. The values of Euclidean distances (d) were first normalized according to the formula:

$$\hat{d}_i = \frac{d_i - {}_{min}d}{{}_{max}d - {}_{min}d}, \quad i = 1, 2, 3, \cdots, n. \tag{34}$$

This provided their variation from 0 to 1 and allowed us to compare this type of calculation with alternative ones. It should be noted that trying measures of similarity other than Euclidean distances did not provide more reliable results.

According to the applied algorithm the next stage was the recognition of uniform groups of territorial units by dividing the resulting ranged series into intervals. This operation can be performed in different ways and allows us to get a number of alternative groupings of territorial units. Final groups were determined by calculating the heterogeneity coefficients that showed the integral degree of difference between territorial units pooled together, as well as by the coefficients of canonical correlation (Tikunov, 1997, a). In both cases — the assessment of environment quality and the assessment of population health — four groups of territorial units were finally differentiated, though calculations were made with their number varying from 2 to 15.

The analysis of obtained results was carried out using corresponding maps and anamorphosises. It appeared that in majority of cases with the help of anamorphosises it is possible to represent the ununiformity of distribution of processes and phenomena under consideration and to estimate the revealed regularities more completely and more visualy than with the help of traditional maps.

Substantial analysis of the results allows us to make several conclusions. First of all, as to the level of environment pollution the whole territory of Russia belongs to the areas of tense ecological states, and even a critical one in some regions and republics. The resulting data confirm the experts' opinion (Losev et al., 1993) that Russia is among the regions contributing significantly to the emergence, persistance and aggravation of negative ecological trends on a global scale. At the same time the complex assessment of unfavourable environmental features has shown that the quality of environment differs greatly over the country's territory, the leading factor being the pollution of such natural components as the atmosphere and the natural waters.

The population health in Russia could be also described as very uneasy: practically all examined parameters were considerably worse than in developed countries. Regional variations of population health correlated to a large extent with the distribution of background pollution, the exceptions being attributed to the complicated and indirect relations in the "human health – environment" system. The least

favourable regions, both in population health and the state of environment, were Kalmykiya, Tchetchen–Ingush Republic, Dagestan, several regions of Siberia and the Far East. The unfavourable ecological situation was characterized by increased values of morbidity and mortality, particularly child mortality caused by congenital anomalies, and also by miscarriages and stillbirths. The threats caused by living in these regions were enhanced by uncomfortable climatic conditions. The most favourable living conditions (natural features combined with relatively low anthropogenic stress) were typical mostly just for the European Russia.

The unfavourable ecological situation can be identified not only by morbidity values and medical demographic parameters, but by the nosological structure of regions as well. Thus the next stage of our study was the regionalization of Russia according to the classes of illness and death causes, both among children and the total population, in 1991.

In particular, the typological classification of ter- ritories was constructed based on the number of cases of the following nosoforms (per 1000 inhabitants): infections and parasitic diseases (1), endocrinic diseases, nutritio- nal, metabolic and immunity disorders (2), diseases of blo- od and blood-forming organs (3), phychosomatic disorders (4), diseases of nervous system and sense organs (5), blo- od-vascular system diseases (6), diseases of respiratory organs (7), digestive system diseases (8), urogenital sys- tem diseases (9), complications during pregnancy, child- birth and postnatal period (10), diseases of skin and sub- cutaneous tissues (11), diseases of the musculoskeletal system and connective tissues (12), congenital anomalies (13), trauma and poisoning cases (14).

The algorithm of multivariance classification is described in the book (Tikunov, 1977, a). Its realization requires the normalization of all initial parameters according to dispersion values and subsequent application of alternative classifications which allow us to divide the set of territorial units into different predetermined numbers of homo-

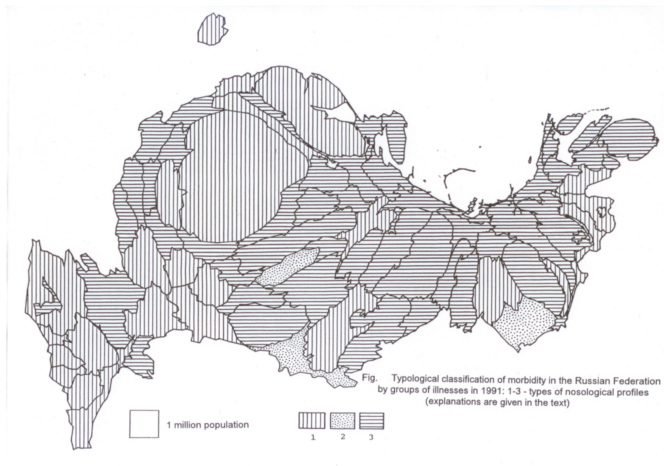


Fig. 81: Types of morbidity of population of Russia.

geneous groups, in this case from 2 to 15. The homogeneity of groups for each alternative classification was evaluated using the mathematical measures of proximity of territorial units. For this purpose there were used the Euclidean distances (see the formula **??**) and the coefficients of correlation in the form of $1 - r$, where r stays for the pair coefficients of the correlation. The best results were achieved by using the Euclidean distances.

The heterogeneity coefficients were calculated for each of 14 resulting groupings with the number of groups varying from 2 to 15 in order to characterize the difference between territorial units pooled together. The analysis of changes of heterogeinity coefficients and the resulting set of alternative typologies allowed us to choose the three-taxon classification as a final one. Arithmetic means of initial parameters were calculated for substantial description of classification units.

Despite the obvious schematizm of the resulting typological classification (Fig. 81) the obtained results allow to come to several conclusions. Thus, according to the data about the classes of illness causes the territory of the Russian Federation could be divided, in fact, into two main taxons characterized by specific nosological profile, or combination of illnesses, and the particular intensity the illnesses manifest themselves with. One more taxon, including Chuvashia, the Orenburg region and the Altai Territory, is of transition nature as to the values of parameters discussed.

The majority of the regions of Russia have been assigned to the second type with higher averages of all parameters under study. Although the diseases of respiratory organs contribute significantly to the pathological structure of all regions of Russia, in this very type, which includes mainly Northern and East-Siberian areas, these diseases are particularly important. Occupational traumatism, diseases of the nervous system and sense organs, as well as infection and parasitic nosoforms are also of great importance there. The first type includes the areas to a certain extent similar in their nosological profile and having lower morbidity values for all classes of illnesses studied. Such territorial units are relatively limited in number (Moscow, Leningrad and Kaliningrad regions, southern regions of Russia and some regions of the Far East).

Mortality values are of substantial importance for describing the relations between the population health and the long-term environmental factors. The analysis of total mortality indices for the years from 1979 till 1988, calculated per 1000 persons according to the 1989 population Census, there was made a typological classification which permits to reflect the main spatial and temporal regularities of distribution of this parameter over the territory of Russia (Fig. 82). The resulting scheme of typological classification by the dynamic of total mortality parameters includes 7 taxon, that can be conventionally grouped into two types with three and four subtypes respectively. Each subtype has a characteristic trend of the total mortality dynamics; all of them are ordered according to the increase of this parameter. Graphs shown on Fig. 83 are the legend for the map shown on Fig. 82.

The first type includes 3 taxons which embrace those areas of Russia where the decrease in mortality has slowed down during the last decade; the taxons differ in the absolute values of parameters discussed. To the second type (four taxons) those areas were assigned where some decline in mortality in the middle 80s was followed by the increase of this parameter towards the end of the decade. The regular-

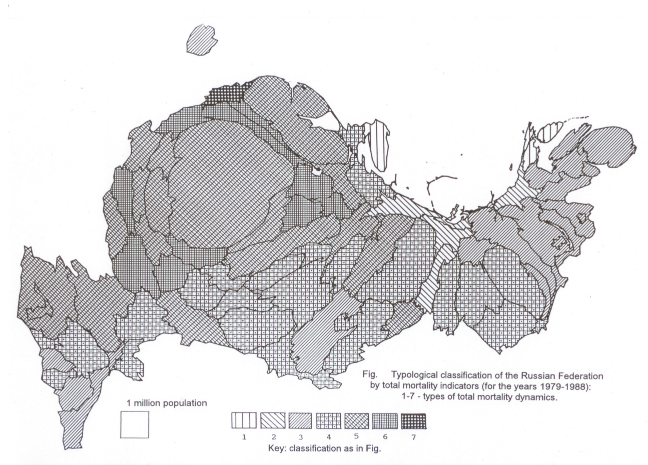


Fig. 82: Types of dynamics of indices of total mortality of population of Russia.

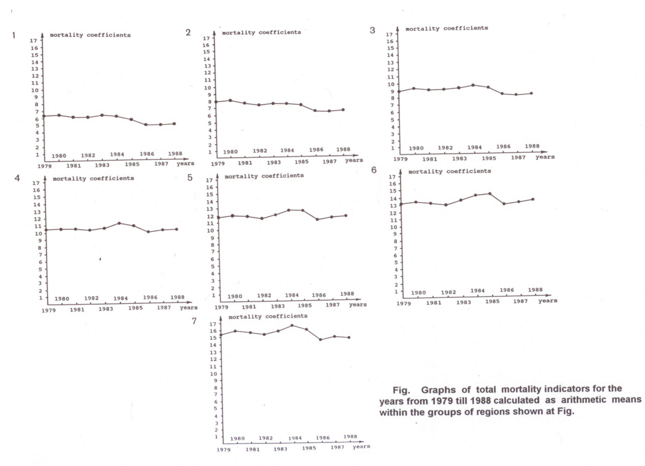


Fig. 83: Graphs of dynamics of the indices of the total population mortality.

ities of spatial and temporal differentiating of the total mortality over Russia have defied unambiguous explanation, they do not obviously correlate with today's environmental situation. At the same time it should engage our attention that practically the whole European Russia and the industrial regions of Southern Siberia are among the areas with the highest mortality rate and the trend exists there for further degradation of the current situation.

Visual analysis of resulting map series allows to conclude that in some cases the anamorphoses give a more clear and comprehensive picture of uneven distribution of processes and phenomena under mapping and a more proper assessment of regularities revealed, if compared with conventional maps. Analysing the territorial component of human environment alone could be inadequate for the assessment of nosogeneity, because the epidemic process as a social phenomenon necessarily correlates with demographic features. In particular, plotting the integral parameters of population health and the environmental factors on an anamorphosis instead of a conventional map allows us to correlate them with population numbers, that, in their turn, focus the attention on those areas where the high values of morbidity and mortality relate to considerable number of people, not to vast and sometimes uninhabited territories.

Let us show the use of anamorphosises for characterization of *interconnections of phenomena* on an example of correlation analysis between the distribution of the alive substance in the ocean and its oil layer pollution. This experimant is a continuation of the example from the previous section and it was described in (Gusein-Zade, Suetova, Tikunov, 1993). As an illustration let us determine correlation connections between between the distribution of the alive substance in the ocean and its oil layer pollution. Logical connection between these two phenomena is very indirect. On one side, in regions where the ocean pollution is higher, there must be less alive substance, though, naturally, there are deviations explaned by other reasons. On the other side, in regions with bigger amount of the alive substance, there are more fishing ships and therefore higher probability of the ocean pollution.

The map of correlations between the distribution of the alive sub-

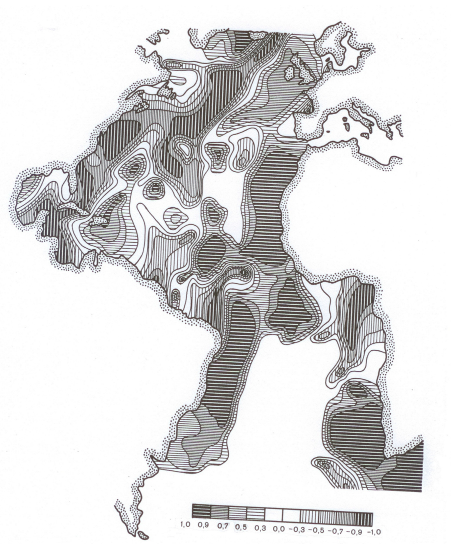


Fig. 84: Correlations between the distribution of the alive substance in the ocean and its oil layer pollution.

stance in the ocean and its oil layer pollution (Fig. 84) was created by the method of "vector" determination of correlation coefficients (Arkhipov et al., 1976, p.83–85), according which the indices were considered as functions on a point of the territory. The correlation coefficient between indices F_1 and F_2 at a point is equal to the cosine of the angle between gradient vectors of the corresponding functions.

To calculate the correlations, values of the indices where taken in vertices of the regular lattice formed by parallels and meridians. For each such vertex there was distinguished the window which included 25 vertices horizontal and vertical coordinates of which differed from the coordinates of the vertex under considerations by not more than two units. Using these 25 vertices (with known values of the indices) with the help of the method of least squares one determined the coefficients of the linear regressions $F_i = a_i x + b_i y + c_i, i = 1, 2$ (here x and y

are interger coordinates of vertices of the lattice). The correlation coefficient at the initial vertex was determined by the formula

$$r = \frac{a_1 a_2 + b_1 b_2}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}}. \tag{35}$$

The analysis of the dependence between the distribution of the alive substance in the ocean and its oil layer pollution shows that in the majority of traditional fishing regions one observes high positive correlations (see Fig. 82). Let us indicate some of them.

1. North seas, namely the south part of the Norwegian Sea, waters around the Iceland are the most productive regions where average annual takes of valuable biological production are equal to 3–5 t/km^2. This also includes the West Europe shelf with the North and Irish Seas, the Denmark Strait, the English Channel and the waters around Great Britain. These are regions of intensive fishing, which originated long ago and played important role

in the economics of coastal countries.

2. The shelf of the North–West Africa is among high productive ones. Average annual takes here are also equal to 3–5 t/km^2 of valuable biological production. Trade–wind currents, which originate at East coasts of the Atlantic Ocean, create in this region driving phenomena, and deep waters rich in biogen elements raise to the surface. The upwelling zone in this region stretches from the Bay of Biscay to the Green Cape Islands. Here one observes the highest (0.9–1.0) positive correlation connections (see Fig. 84).

3. Highly productive region of tipical upwelling and intensive sea fishing near the coasts of the South–West Africa (between the latitude 17^o South and the Cape of Good Hope). Annual takes here are from 3 to 5 t/km^2 and more, the correlation dependence between the distribution of the alive substance in the ocean and its oil layer pollution is positive and high (0.9–1.0).

4. The shelf of the South America, which includes the shelf of Brazil the La Plata Bay and the Falkland–Patagonia shoal. This region is also characterized by high positive correlation connections (0.7-1.0), by its average annual take (3-5 t/km^2 and more) of valuable biological production it can be compared with the zones of upwelling of the

North–West and South–West Africa.
5. The region of open ocean between the North and the South trade–wind currents. In the northern hemisphere it includes Southern parts of the Canary and the Antille currents, in the southern hemisphere it includes Northern parts of the Bengal and Brazil currents. The region lies between the tropics, the Lomonosov current divides it into almost equal parts: the North and the South ones, what leads to the distribution of geochemical elements different from that in other lowly productive tropical waters. This tropical region of the Atlantic Ocean is attributed to productive zones of the World Ocean, average annual takes here are from 1.5 to 3 t/km^2, the correlation connections in the most part of it reach 0.7-1.0 (see Fig. 84).

Thus, the described regions of the Atlantic Ocean are highly productive, abundance of the nekton (valuable production) in them correspons to main regularities of distribution of alive substance (phyto-, zooplankton, and benthos). High positive correlation dependences between the distribution of the alive substance in the ocean and its oil layer pollution is also confirmed by literature data (Bulatov et al., 1977; Atlantic ... , 1984).

However, far from all regions of the Atlantic Ocean known as rich in fish have positive correlation estimates of the dependences between the distribution of the alive substance and the oil layer pollution. Among them it is necessary to draw attention to the North–West sector of the Atlantic Ocean, which includes the Newfoundland shoal, known for the mankind for more than 500 years, and also the Labrador region, the shelfs of the New Scotland and of the New England, visiting by flotillas of many European countries. On the map (see Fig. 84) high negative correlation coefficiets (from -0.7 to -1.0) are observed in these regions.

Other regions with high negative correlation dependences are the shelf of the North America between Florida and the Cape Cod, the slope of the shelf washed by the Gulf Stream. These regions are important fishing objects with maximal average annual takes (3–5 t/km^2).

As the map shows (see Fig. 84), not high negative and zero correlation dependences have aquatories of the Atlantic Ocean, where the biological mass of the alive substance is respectively small, average annual takes of valuable production reach 1,0–1,5 t/km^2 (Atlantic ..., 1977). These aquatories of the Gulf and of the North–Atlantic Streams, of the Saragossa Sea, i.e. regions of the anti-cyclonic rotation between the Noth trade–wind current and the Gulf Stream (between the latitudes 25^o and 35^o South and the longitudes 40^o and 75^o West), and also central regions of the Caribbean Sea and of the Gulf of Mexico, where values of average annual takes are even smaller (0,3–0,5 t/km^2).

An analysis of the map of correlations between the distribution of the alive substance in the ocean and its oil layer pollution does not permit to come to definite conclusions, e.g., to confirm that more polluted by the oil layer regions correspond to aquatories more rich in the alive substance. Thus the general conclusion is that the map of correlations reflects some regularities, however in some cases they are not observed or it is difficult to explane them. This forces us to turn to the use of an anamorphosis.

For a continuation of the experiment we used such an index as the density of fishing ships in the ocean, which is connected both with the distribution of the alive substance and with the pollutionof the ocean by the oil layer, i.e., we tried to evaluate the connection between two indices through a third one. The distribution of the alive substance and the oil layer pollution were put on the anamorphosis based on the distribution of ships

in the ocean (Fig. 85 and 79 respectively). After that the correlation coefficients were determined by the above described method. The new map of correlations between the distribution of the alive substance in the ocean and its oil layer pollution is shown on Fig. 86. Here correlation connections between the phenomena can be seen more clearly than on Fig. 84 what can be illustrated by the following examples:
1) there became apparent the regularity which confirms the presence of high correlation dependences between the amount of the biological mass in the ocean and its oil leyer pollution in highly productive circum continental regions compared with open waters of the ocean;
2) the most high positive correlation dependences are observed in zones of the upwelling and in traditional fishing regions: near the coasts of the South–West and the North–West Africa, of the South America,

Fig. 85: Distribution of the alive substance in the ocean, kg/m^3, on the anamorphosis, based on the density of ships in the Ocean.

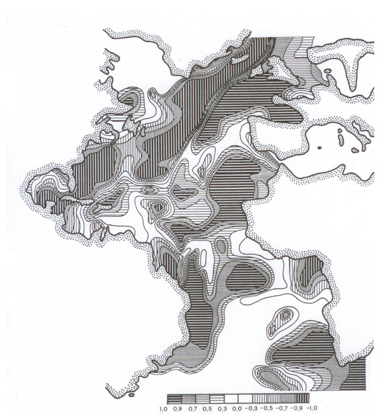


Fig. 86: Correlations between the distribution of the alive substance in the ocean and its oil layer pollution on the anamorphosis.

and also South from the latitute 20^o South;
3) there became distinguished the region of the Newfoundland shoal, as a highly productive aquatoria with high positive correlation connections;
4) there became more definite boundaries of traditional highly productive fishing regions, such as the shelf of the West Europe, the North–West part of the Atlantic Ocean, Northern Seas (see Fig. 84);
5) there became very apparent lowly productive regions, situated in the central rotations of the tropical zone, which have zero and negative values of correlation dependences, which is more visually reflected in the southern hemisphere.

Therefore, one can assert, that after introducing into a cartographic image a parameter connected with the indices under consideration connections between these indices become seen more clearly and logically. Such an approach somewhat resembles the use of partial correlation coefficients, but it solves the problem in graphic form, more convenient for the space analysis.

As an example of the correlation analysis of substancial characteristics of phenomena let us consider indices of people health foe countries of the world (Malkhazova, Tikunov, 1993, a). At present the medical geography pays considerable attention to the study of regularities of influence of natural and social conditions of concrete territories on the health of the population. It is possible to insist that even potentially effective profilactic arrangements will not be succesful if they do not take into account data on spatial differences in spreading of diseases.

Medical-geographical territorial differentiation is studied with the help of various approaches and methods. Analysis of existent schemes of regionalization, which reflect results of use of these approaches, permits to note that objects of regionalization more often are either individual characteristics of the environment or individual nozological units (Raykh, 1984).

The object of the described study is to reveal spatial differences in the health of population of the world by a complex of most characteristic natural-endemic diseases. These diseases include infections and invasions, the pathogenes and carriers of which are members of natural ecosystems, i.e. parasitic systems of such diseases can be regarded as a biotic component, organically bound with natural ecosystems.

It is known that transmission and diffusion of natural-endemic diseases is possible in definite natural conditions. According to this basic idea the study of medical-geographical territorial differentiation of the Earth can be carried out by the number of natural-endemic diseases and the peculiarities of their spreading. Differences in conditions are tied, in the first place, with the climatic zonation of the Earth. Therefore the determination of characteristic natural-endemic diseases for investigations of basic regularities of their territorial diversity and geographical distribution was carried out for three main natural zones: torrid, temperate and cold.

For an epidemiological characterization of these zones the paper Raykh, Maksimova (1988) has been used. There is based on bibliographical data and expert estimations description of characteristic natural-endemic diseases for these zones and information about their geographical distribution. For the torrid zone characteristic diseases are Malaria, Visceral leishmaniasis, Cutaneous leishmaniasis, Sleeping sickness,Chagas disease, Ancylostomidoses (Hookworn disease), Necator americanus, Strongylid threadworm, Onchocerciasis, Wuchereriosis, Filariasis Malay, Loaosis, Dracunculiasis, Schstosomiasis intestinal, Schstosomiasis urinary, Schstosomiasis japonicum, Liver fluke, Lung fluke (Paraganimose), Yellow fever. For the temperate zone characteristic are a series of Arthropode borne viruses - Eastern equine encephalomyelitis, Western equine encephalomyelitis, Colorado tick fever, Saint Louis encephalitis, tick-borne encephalitis, hemorrhagic fever, rabies, Asian tick-borne rickettsiosis, Leptospirosis grippotyphosa, Leptospirosis pomona, Leptospirosis icterohaemorrhagiae, Tularemia. In the cold zone the most characteristic natural-endemic diseases are Alveococcosus, Brucellosis, Yersiniosis.

Since the bulk of the data on sick rate of the population has to do with units of administrative division of a territory, the political map of the world was use as an initial working base. Territories of countries were used as operational territorial units for calculations.

There were data on sick rates of 34 natural-endemic diseases listed above for all countries of the world. The following scale has been used: 3 - the disease is spread in all parts of the country; 2 - the disease can

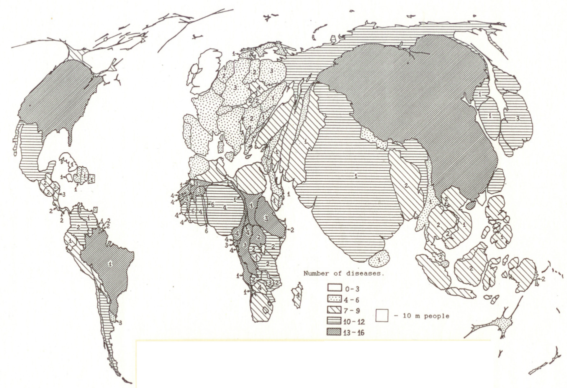


Fig. 87: Total number of nature-endemic diseases of population of countries. Digits show the number of diseases spread everywhere over the territory of the country. (The anamorphosis of the World is compiled on the base of the numbers of population of the countries.

be met in some regions of the country; 1 - the disease can be found in some nidi, 0 - the disease can not be met in the country. These grades were taken from the paper (Raykh, Maksimova, 1988). The most part of data was for the 1989. Therefore boundaries of countries are for this date too: there are shown territories of the former Soviet Union, of Yugoslavia and of DDR.

For the characterization of differences between countries by sets of naturalendemic diseases there was used the method of correlation analysis with the representation of its results in anamorphosises - graphical images obtained from traditional maps, the scale of which is not constant and changes in accordance with the value of an index, on which it is based. The method of compilation of anamorphosises has been described in (Gusein-Zade, Tikunov, 1990). Anamorphosises can serve as cartographical bases for more visual representation of medical-geographical characteristics.

Even simple putting initial data on a map permits to observe some regularities. In the Fig. 87 there is shown an anamorphosis based on the data on numbers of population of countries of the world (i.e., the areas of country images are proportional to the number of population: see Fig. **??**) with the information about the total numbers of natural-endemic diseases for countries in a five-stepped scale. The increase of intensity of shading means the deterioration of conditions of the environment, connected with the increase of the total number of diseases. In addition by figures there is indicated the number of diseases spread in all parts of a country. Absence of a number in a country testifies for the absence of widespread diseases in this territory (absence of the figure means "0").

Putting all these data not on the traditional map, but on the anamorphosis permits to correspond them visually with numbers of population of countries and to attract attention of readers (for example of health services) to territories where high sick-rate concerns not big and sometimes not populated areas, but considerable masses of population. Moreover, the analysis only of the natural component of the environment of a human being for an estimation of a potential possibility of an occurrence of a natural-endemic disease in a territory can be not sufficient, because an epidemic process as a social phenomenon is necessarily connected with the population. In such cases anamorphosises reflect simultaneously both the estimation of natural preconditions and possibility of their realization. Therefore they give possibility to analyze natural factors together with social ones.

Visual analysis of the anamorphosis permits to notice a series of regularities in the spreading of natural-endemic diseases over the territory of the Earth. From the Fig. 87 it can be seen that total number of natural-endemic diseases in countries changes from 0 to 16, the number of diseases spread everywhere in a territory of a country changes from 0 to 6. Apparently, the role of preconditions of characteristic natural- endemic diseases in the global scale plays the geographical zonation of the natural environment. As a rule the number of characteristic natural-endemic diseases, which are most closely connected with the biota, decreases from the torrid zone to the cold one. This regularity is more clearly observable in the territory of the Old World. The obtained conclusion conforms to the known scientific fact: complexity and variety of the structure of torrid ecosystems is much more bigger, than of cold ones.

The abundance of natural-endemic diseases in the torrid zone side by side with general spreading of them aggravates the extremeness of the natural environment in countries, situated there. A widespread phenomenon in this territory is the polyparasitism, because the high intensity of infection of torrid diseases increases the probability of simultaneous infection by several types of pathogenes. Many natural-endemic diseases, which are spread sporadically in natural conditions, can cause large epidemic outbreak after getting into a human collective. Therefore a particular problem for public health services are developing countries , in which a considerable number of existing diseases goes with a high density of population. as can be judged by the anamorphosis (Fig. 87) in the first place it takes place in China, in India, in a number of countries of Africa, in Brazil.

For the further investigation of spatial differences in the spreading of natural-endemic diseases this anamorphosis has been analyzed side by side with other analogous images, which characterize values of correlation coefficients between pairs of countries. It has been used a method of putting into a map of correlation coefficients which were calculated on the base of so named Q-modification of the correlation analysis (Zhukov, Tikunov, 1977). In this case correlation coefficients are determined for each pair of territorial units, characterized by long enough series of homogeneous, mono-structural data. The length of the series (the number of indices, characterizing territorial units) have to guaranty enough reliability of calculation of correlation coefficients.

In this case for all pairs of countries there were determined correlations coefficients by sets of characteristic natural-endemic diseases with taking into consideration of intensity of displaying of each nozological unit. The results of the correlation analysis were put in anamorphosises too. As examples in Fig. 88, 89, and 90 there are shown three of them, which describe the comparison of series of diseases in all countries of the world with respect to the former USSR, To India and to Zaire correspondingly. It means that this pictures reflect correlation coefficients of epidemic situation by natural-endemic diseases in the USSR, in India and in Zaire with analogous series for all other countries of the world.

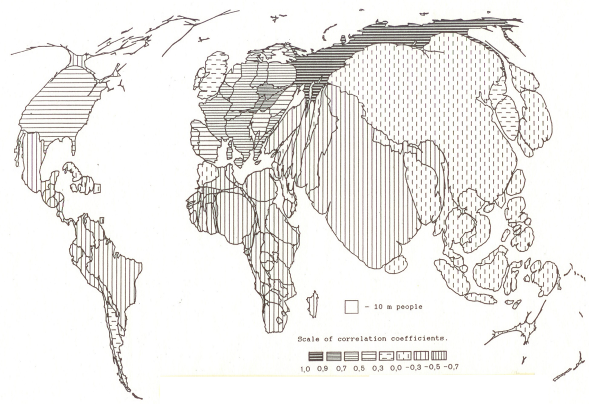


Fig. 88: Correlation coefficients of appearing of nature-endemic diseases in the former USSR with other countries of the World.

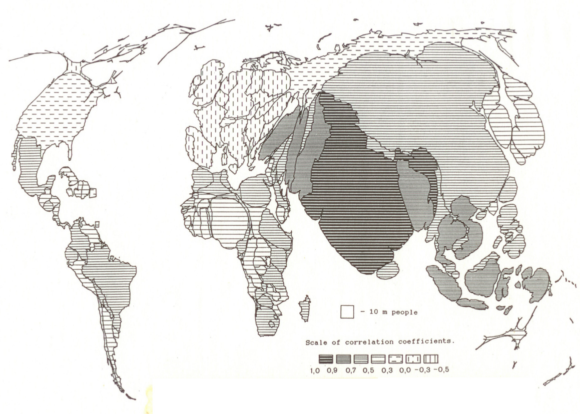


Fig. 89: Correlation coefficients of appearing of nature-endemic diseases in India with other countries of the World.

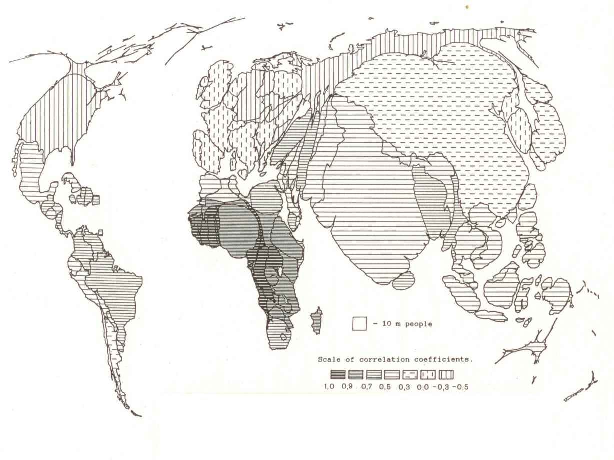


Fig. 90: Correlation coefficients of appearing of nature-endemic diseases in Zaire with other countries of the World.

The obtained variation of coefficients is rather simple and easily explainable. For example the Fig. 88 testifies for the most similarity of the epidemic situation in the territory of the former USSR with the situation with natural-endemic diseases in Austria, in Hungary and in Czechoslovakia. The next by the value of correlation coefficients are majority of other countries of Europe. Then Canada follows. There is not observable connection of the set and intensity of natural-endemic diseases in the territory of the USSR with China, countries of the South-Eastern Asia and Australia. It is possible to say about complete dissimilarity in the epidemic situation in the USSR in comparison with the Arabian world, India, Latin America and especially with Africa. Conclusion of analogous type can be done by the analysis of Fig. 89 and 90.

Meaningful analysis of different subjects shown in figures permits to obtain additional information about peculiarities of spreading of natural-endemic diseases in countries of the world. In particular, it is possible to say about clearly expressed medical geographical specific character of individual continents. Tracing the change of the epidemic situation along the line of the equator, it is possible to notice that, despite of respectively insignificant changes of coefficients of similarity between countries by sets of diseases and intensity of their displaying, the Chagas disease is characteristic only for the South America, the Sleeping sickness — for Africa, the Schistosomiasis japonicum can be met only in Asia. Even more clearly regional peculiarities are expressed in the temperate zone. For example along the profile, which goes approximately at latitude 50^o North, it is possible to notice that diseases, caused by viruses of Eastern and Western equine encephalomyelitis, Colorado tick fever, virus of Saint Louis encephalitis are spread only in the American continent, the Tick-borne encephalitis in the Eurasia, the Asian tick-borne rickettsiosis of the North Asia - in Asia only.

It seems that one of possible reasons for specific medical geographical character of continents is connected with the influence of peculiarities of development of the flora and the fauna on the forming of parasitic systems. Maximal similarity by the set of characteristic natural-endemic diseases has territories which belong to same biomes - Nearctic, Neotropical, Palearctic, Afrotropical, Indomalayan, Oceanian, Australian.

The analysis of data shows that differences between continents are also expressed in the character of relation of indices under consideration — total number of characteristic natural–endemic diseases and the number of diseases, spread everywhere. For example in Africa the increase of total number of characteristic natural-endemic diseases takes place with the increase of number of diseases, spread everywhere. In South and Central America such a regularity can not be observed. Here the number of diseases, spread everywhere, is not large and practically does not change with the increase of total number of diseases. However it is possible that it is connected with big influence of social–economic factors in Central and South America, what has been expressed in decrease of the number of spread everywhere natural-endemic diseases of the population.

Limited size the section do not permit to dwell on other aspects of spatial differences in medical–geographical situation in the world with respect to the complex of the natural-endemic diseases. Let us only notice, that offered approaches to the study of spatial differentiation of a territory can be used for the analysis of differences in health level

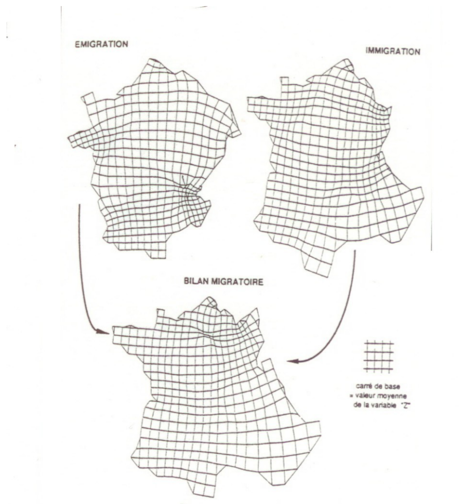


Fig. 91: Anamorphosises which show the migration in France in 1975–82 (after Cauvin, Enaux, 1991).

of population for different diseases and characteristics of the environment.

For depicting characteristic of *dynamics* on anamorphosises let us use an example of mapping forecasted values of number of population of countries of the World. Having data on the population numbers for a series of year and analysing tendencies in time series, demographers make a forecast for several years ahead. Such forecasts are rather widespread and they can be used as bases for corresponding anamorphosises. ¿From variety of examples one can mention images which show the migration in France in 1975–82 (Cauvin, Enaux, 1991), Fig. 91 and the growth of the world population from 1750 to 2000 (Cole, 1979): Fig. 92.

Let us give also a series of anamorphosises compiled by the authors on the base of data of the Department of analysis of economics, social

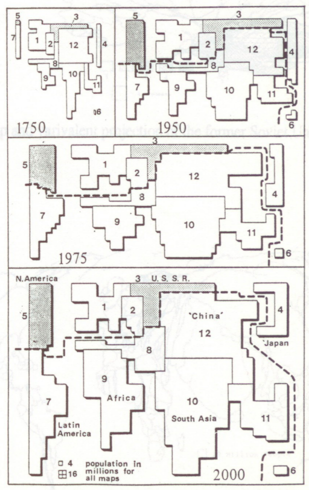


Fig. 92: The growth of the world population (after Cole, 1979).

information and politics of U.N.O.: actual and values of number of population of countries of the World for 1950-2050 (Fig. 93).

The sequence of anamorphosis shows changes in time and they can be considered as separate time slices of the process.

Let us give also a series of anamorphosises compiled on the base of data on Gross National Product for 2003, 2015, and 2020 (Fig. 3, 4, 5 of the coloured inset), on the data on numbers of population of the age up to 14 years for the same years (Fig. 6, 7, 8 of the coloured inset; the anamorphsises were compiled for and were based on data of the World Bank) and a number of other interesting geographical characteristics (Fig. 9 of the coloured inset and so on) [3].

V.3. Complex geographical analysis of phenomena shown on anamorphosises.

As an example let us consider evaluation of regions of Russia in

[3] *Anamorphosises on the coloured inset starting from Fig. 3 were compiled for GRID Arendal*

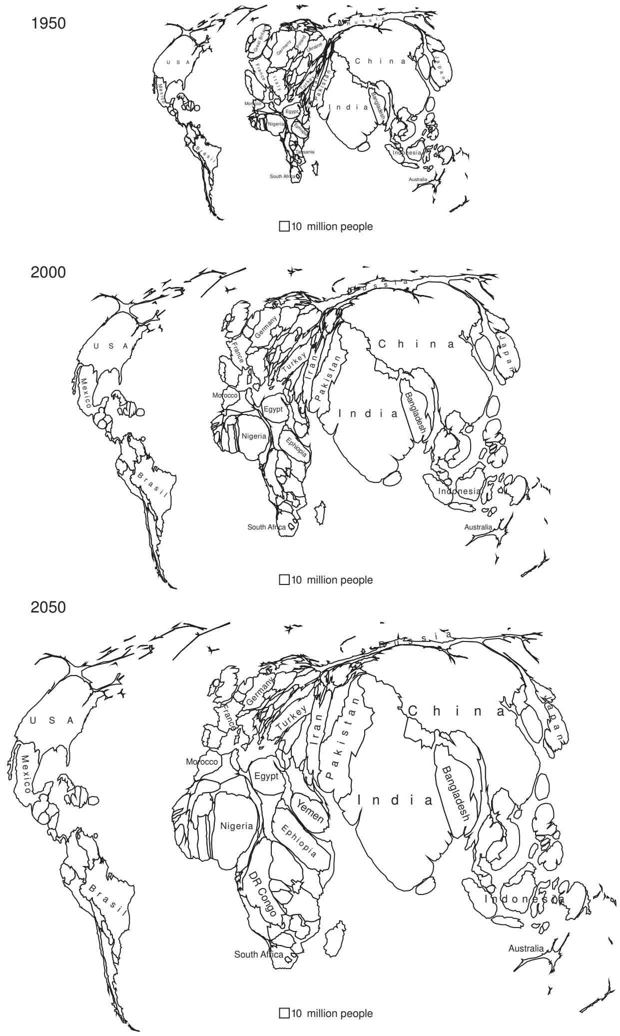


Fig. 93: Anamorphosises of the World compiled on the base of numbers of population of countries for 1950, 2000, and 2050.

three interconnected directions: health services, living standards, and social- economic development and let us visualize the obtained results with the help of anamorphosises (N.A.Borodulina, V.S.Tikunov, 1998). The indices to characterize a particular region have been chosen on account of their importance in the formation of budgetary expenditures of the particular region. The results obtained are presented in the form of a classification of regions and can be recommended as background to elaborate objective estimation of regions' budgetary needs and thus, a system of minimal state standards.

The problem of estimation of standard budgetary needs appears to be one of the most sophisticated in building Russia's budgetary federalism. In legislation on subventions and budgetary rights of the component units of the federation, attempts were made to use federal guaranties on the state expenditures intended directly for particular regions, minimal budget and cost norms, in solving the problem in question. But unfortunately, the procedures proposed remained just proposals and no concrete calculations have been made or budgetary planning and regulation mechanisms assuming the new ideas have been developed.

Social norms, responsible for the level of budget-supplied social well-being are now obsolete and in no way correspond to the new reality, the fact confirmed by our studies in the sphere of public health. Besides, territorial budgets have received new sources of revenues. The whole picture is different now with the social infrastructure of facilities themselves: on the one hand, managing and maintaining have become cheaper; on the other hand, many enterprises have started to pay increased wages so as their workers could satisfy their social needs individually. The well-to-do people have created their own social infrastructure, instead of using municipal one. All these changes call for re-estimation of financial needs of a particular region and of what it can afford. It has become impossible to implement control over the situation and thus, form budgetary expenditures, using the old norms.

One conclusion must be made, that it is completely impossible to estimate regions' needs (including budgetary ones) without taking into account the specific character of each region, peculiarities, its differences from other ones. And in this context, the importance of the problem under consideration can hardly be overestimated.

It is in no way fortuitous that the focus of the study is the state of affairs in public health, and it is easy to understand why. Health of a whole nation depends on health of a particular individual, and, following the definition of the World Health Organisation (of the United Nations), health is the state of complete physical, spiritual and social wellbeing. Therefore this sphere is most sensitive to social and economic changes in a country. Moreover, budgetary expenditures for public health account for one forth of a region's budget, which is not little. It is of convenience that the statistic materials on regions available allow using in our analyses those data which are of particular significance to characterize the state of affairs in the sphere of interest.

he indices selected for calculations had to satisfy the following requirements:

1. They must effect changes in budgetary expenditures.
2. Every following set of indices must include the preceding set so as to determine the effect of the indices added and to compare the more concrete typologies with the more general ones. To clarify, the commanding view of public health can be investigated against the background of the standard of living and the level of social-economic development of particular regions of Russia.

The calculations did not include data on Chechnya and Ingushetia because of the lack of statistics on many indices concerned.

Most of analysis was based on data for 1994, that is for the very period of developing budgetary federalism and establishing the Federal Fund of financial support of regions. Calculations of the present accounting for each particular component unit of the Federation in Fund–1994 and

Fund–1995 were made using data for 1993. Some indices (e.g., decline in production) required analysis of data for a longer period of time (1990–1994).

To estimate health of the population and the level of public health care development in the regions of Russia, two groups of indices characterizing health of the population of a particular region have been chosen. The first group involves 11 indices characterizing health of the population of a particular region. The second group of 8 indices concerns public health care and environmental conditions in Russia. So, indices of the first group show morbidity with some of the most representative diseases, the spreading of narcotism and alcoholism. Besides these, the first group also involves the following three demographic indices: infant mortality (which reveals availability of medical aid), mortality at the most able age and the life expectation at birth (the latter signalling the various impacts and appearing to be of most importance for a comprehensive estimate).

Health level of the population

1. The infant mortality per 1000 people;
2. Morbidity of population per 1000 people;
3. Morbidity with infectious and parasitic diseases;
4. Morbidity with malignant neoplasms;
5. Morbidity with the blood circulation system;
6. Morbidity with the respiration system;
7. The number of people registered due to alcohol abuse per 1000 people;
8. Those registered as practising drug abuse per 1000 people;
9. Those misusing other substances per 1000 people;
10. Mortality of people at an able age per 1000 of people of the same age;
11. Life expectation at birth.

The second group of 8 indices characterizes the development of the public health care and the condition of the environment in Russia. The group involves indices describing social infrastructure in the field of public health care, investment thereunto and the two most important ecological indices, namely intake of pollutants with waste waters into water reservoirs and air pollution with contaminants.

The level of public health care development and the condition of the environment

1. Number of physicians engaged in the medical care of population per 10000 people;
2. Number of hospital beds provided to population, per 10000 people;
3. The ratio of the number of maternity consultations, and children's polyclinics to the sum of the population under the able age and to the female population;

4. Number of hospital beds provided to children, per 10000 children;
5. Air pollution with contaminants;
6. Intake of pollutants with waste waters into water reservoirs;
7. The percentage of investments into public health care in all sorts of investments;
8. The reception capacity of polyclinics and other medical facilities rendering out-patien and dispensary services to population (for one shift) per 10000 people.

The following group of indices depicts the standard of living in Russia (Dmitrijeva, 1992). Besides the 19 indices of the above two groups, it contains another 22 indices, 4 indices responsible for the incomes of the population; 2 indices characterizing the situation with migrations and refugees; another two accounting for the level of unemployment; and 6 indices showing the level of education, the rest revealing social infrastructure, criminal situation, national income distribution.

The standard of living

A. Indices characterizing health of the population.
B. Indices characterizing public health care.

To add:

1. The ratio of an individual income to the cost of living, %;
2. The number of the poor to the population of the particular region, %;
3. The number of the cost of the main 19 food products to the income, %;
4. The ratio of per capita income growth rate to the price growth rate, %;
5. Migration growth per 1000 people;
6. The number of refugees in the regions of Russia, per 1000 people;
7. Volume of marketable services rendered to households, per capita, in billions of roubles;
8. Per capita retail trade turnover, in billions of roubles;
9. The ratio of the registered unemployed to the economically-active population, %;
10. Rivals for one vacancy;
11. Number of crimes reported, per capita;

12. National income per capita, in thousands of roubles;
13. Number of home telephone devices in operation of ordinary exchange or connected to them available to urban inhabitants, per 100 families;
14. Number of home telephone devices in operation of ordinary exchanges or connected to those, available to rural inhabitants, per 100 families;
15. Number of personal passenger motor cars owned by households, per 1000 people;
16. Housing provided to households, per capita;
17. The ratio of the number of students of higher school to the able age population, per 10000 people;
18. Share of enrolment in State general education day-time schools with classes in the second shift, in total enrolment, %;
19. The ratio of pre-school institutions to the number of children of the corresponding age, %;
20. The percentage of investments into education in all sorts of investments;
21. The percentage of working specialists with higher education;
22. The percentage of working specialists having incomplete secondary education and those, without education.

The last group of indices, which is the largest one, characterizes the level of social-economic development of Russia's regions. Besides the 41 indices showing the standard of living, this group includes another 7 points accounting for the economic development of the regions. It should be stressed here that three of them are complex and embrace a number of indices belonging to a particular economic kraj (Electronic Atlas..., 1995). For instance, to estimate unprofitableness of a region's economy, use was made of the Goskomstat data on the ratio of the percentage of unprofitable facilities of all industries and the amount of losses to the calculated volume of conventional pure product. Also, account was taken of working time losses in industry (data on other branches unavailable).

To show investing attractiveness, data were applied on per capita investments and the percent of non-centralized investments (excluding federal ones).

To estimate the export index, account was taken of per-capita export from the regions, the percent of hard currency receipts, the percent of export industrial products and the fact if the juridical persons had foreign currency on their accounts.

Other indices characterize changes in the physical volume of industrial and agricultural production, foreign investments into Russia's economy and profits in all spheres thereof.

The level of social-economic development

A. The standard of living.

Also, the level of economic development:

1. Index of dynamics of the physical volume of industrial production, 1994–1995;
2. Index of dynamics of the physical volume of agricultural production, 1994–1995;
3. Index of unprofitableness of a region's economy, 1994–1995;
4. Index of investing attractiveness, 1993–1995;
5. The per-capita volume of production of enterprises with share participation of foreign investors;
6. Export index;
7. The ratio of total profit (in all spheres of economy) to the number of those employed in economy.

Thus there were chosen 48 indices, including synthetic ones. To make all the initial indices compatible they were normalized using standard formulae.

Comparing all the indices for the administrative–territorial units (ATU) to a reference unit characterized by ($\overset{\circ}{x}$) (see formula 33), the ATU indices were ranged with respect to the condition of health (1), the level of public health care (2), the standard of living (3) and the level of social-economic development (4). In the procedure use was made of the Euclidean distances (d) to measure the proximity of the cells to the reference unit having the worst available values of all the indices ($\overset{\circ}{x}$) concerned.

The procedure of dividing the initial set of ATU into distinctly outlined groups had the following steps. First, each of the rated rows was divided into homogenous segments using the algorithm derived earlier (Tikunov, 1995). The number of such segments containing homogenous groups of ATU was reducing sequentially in the course of multi-variant calculations from 15 to 2 and, with the help of special , heterogeneity coefficients, the statistically optimal number of the ATU groups was determined for each particular case. At this stage, the result was conventional grouping of the ATU, with the latter strictly belonging to one or another group. But there was a feeling that in each group there ought to be a kind of "nucleus" with some "halo" around.

Now, to determine the "nuclei" of the groups the following steps were made. First, a reference ATU was obtained, our term denoting some hypothetical ATU which parameters would be mean values of all the actual indices of all the actual ATUs forming a particular group. Then Euclidean distances were calculated from the reference ATU to all the actual 87 ATUs under study. This procedure was repeated for every group as if examining each group in turn. (The number of groups varied, for instance, in the case of social-economic estimation, 6 groups were identified and thus, 6 rows of Euclidean distances calculated. The same procedures were performed to estimate standard of living etc.)

It seemed unreasonable to identify more than one "nucleus" and one "halo" in one group. This raised the problem of dividing each rated row into the following parts: "the nucleus" including the most homogenous ATU group with the least available Euclidean distances to the reference centre; "the halo" zone, and those ATUs which show low attraction to the corresponding centre. In other words, for each of the 6 (for the social–economic estimation) rated rows, our task was to find 2 homogenous parts and "the residue" accounting for the ATUs untypical of the particular group under examination. This task can be solved using a modification of the estimation algorithm. In our experiment we applied the technique of identifying the uniform d–values by means of determining the greatest gaps in the rated rows (Tikunov, 1985). Note, that besides the homogenous "halo", the rated row could contain more than one "residue". Therefore, the dividing of each particular row required multi-variant algorithm procedure, with the number of "residues" ranging from 10 to 2, and the statistically optimal number thereof was calculated as at the first stage.

Fig. 94: Classification of health condition of the population.

It should be mentioned, however, that in this case only “the nucleus” and “the halo” of the row were analyzed, the remaining ATUs left out of account. The result for the 6 rows are to be found in Table 2, with the “nuclei” shifted to the left and “the halos” shifted to the right.

Besides being presented in tables, the results are also shown on anamorphoseses built using the values of population density of a particular administrative-territorial unit (ATU), where the areas of the territorial units on the anamorphoseses correspond to the population of the ATUs concerned. The “nuclei” of the classifications are presented on the anamorphoseses as solid cross-hatchings with the orientation and distance between lines varying depending on integral characteristics becoming better, while the “halos” are plotted with dotted cross-hatchings with the orientation and distance between lines

looking as “mean values” with respect to the corresponding “nuclei”.

Fig. 94 presents the picture of health condition of the population of Russia’s regions. In view of the fact that the Russia health standards are much lower as compared to the Western ones, the classification involved the following 5 categories: 1) deep crisis; 2) very low level; 3) low level; 4) lowered level; 5) medium level. The categories presenting higher standards of health simply lacking in Russia.

The other classifications also involved the same 5 or 6 categories: 1) deep crisis; 2) very low level; 3) low level; 4) lowered level; 5) medium level, 6) relatively good level.

The first group (deep crisis) comprises the Northern krajs of Russia (the Komy Republic, Tomskaya oblast’, Magadanskaya oblast’, Chukotskaya oblast’, etc.). These territories are characterized by harsh climate, polluted environment, high concentrations of male population, wide-spread alcoholism, high infant mortality, great number of ontological patients, shorter life-time. Ecologically unsafe regions include, first of all, Magadanskaya oblast’ with its mining industry and

Classification of levels of the population health		Classification of levels of health services and of the state of the environement from the point of view of its influence on the population health	
Kernels	Auras	Kernels	Auras
			Leningrad oblast
1st group			Khanty-Mansi AR
57. Kemerovo oblast			
5. Komi Rep.		1st group	
60. Tomsk oblast		Sverdlovsk oblast	
	71a. Koryak AR	Rostov oblast	
	72. Magadan oblast	Kemerovo oblast	
	74. Sakha (Yakut) Rep.	Dagestan Rep.	
	62b. Evenk AR		
	71. Kamchatka oblast		
	4. Karel Rep.	2nd group	
	72a. Chukotka AR	Orenburg oblast	
	67. Tyva Rep.	Krasnodar Kray	
	55. Altai Kray	Chelyabinsk oblast	
	73. Sakhalin oblast	Stavropol Kray	
		Murmansk oblast	
2nd group		Samara oblast	
64a. Ust–Ordyn AR		Irkutsk oblast	
48. Kurgan oblast			Vologda oblast
1. Arkhangelsk oblast			Bashkir Rep.
37. Kalmyk Rep.			Yamal-Nenets AR
62. Krasnoyarsk Kray			Tatar Rep.
56. Altai Rep.			Moscow oblast
64. Irkutsk oblast			Komi Rep.
1a. Nenets AR			Nizhni Novgorod oblast
54. Udmurt Rep.			Perm oblast
31. Astrakhan oblast			Penza oblast
62a. Taimir AR			Karachai–Cherkess Rep.

Table 2:

Kernels Auras	Kernels Auras
61a. Khanty–Mansi AR	
3. Murmansk oblast	3rd group
61. Tyumen oblast	Kostroma oblast
61b. Yamal–Nenets AR	Tver oblast
69. Khabarovsk Kray	Kaluga oblast
23. Mariy-El Rep.	Lipetsk oblast
St Petersburg	Tula oblast
68. Primorye Kray	Buryat Rep.
	Tyumen oblast
3rd group	Ulyanovsk oblast
50a. Komi–Perm AR	Primorye Kray
21. Nizhni Novgorod oblast	Kurgan oblast
69a. Jewish AR	Novosibirsk oblast
58. Novosibirsk oblast	Bryansk oblast
7. Novgorod oblast	Krasnoyarsk Kray
Moscow	Saratov oblast
50. Perm oblast	Ust–Ordyn AR
33. Samara oblast	Altai Kray
49. Orenburg oblast	Tambov oblast
36. Ulyanovsk oblast	Voronezh oblast
34. Penza oblast	Orel oblast
51. Sverdlovsk oblast	Adigei Rep.
2. Vologda oblast	Volgograd oblast
52. Chelyabinsk oblast	Ryazan oblast
43. Rostov oblast	Belgorod oblast
12. Tver oblast	Arkhangelsk oblast
17. Ryazan oblast	Omsk oblast
18. Smolensk oblast	Kursk oblast
6. Leningrad oblast	Smolensk oblast
16. Orel oblast	Udmyrt Rep.
20. Yaroslavl oblast	Pskov oblast
15. Moscow oblast	Chuvash Rep.
59. Omsk oblast	Kaliningrad oblast
8. Pskov oblast	Komi–Perm AR
9. Bryansk oblast	

Table 2: Continuation.

Kernels Auras	Kernels Auras
10. Vladimir oblast	4th group
63. Khakass Rep.	Vladimir oblast
19. Tula oblast	Yaroslavl oblast
	Tomsk oblast
4th group	Chita oblast
14. Kostroma oblast	Mordovian Rep.
35. Saratov oblast	Taimir AR
75. Kaliningrad oblast	Kabardino–Balkarsk Rep.
40. Adigei Rep.	Aginsk–Buryat AR
26. Belgorod oblast	Mari Rep.
11. Ivanovo oblast	Kirov oblast
30. Tambov oblast	Jewish AR
22. Kirov oblast	Khabarovsk Krayy
13. Kaluga oblast	Karelian Rep.
65. Chita oblast	North Ossetian Rep.
38. Tatar Rep.	Kalmyk Rep.
25. Chuvash Rep.	Sakhalin oblast
53. Bashkir Rep.	Novgorod oblast
39. Krasnodar Kray	St Petersburg
70. Amur oblast	Khakass Rep.
32. Volgograd oblast	Amur oblast
66. Buryat Rep.	Moscow
29. Lipetsk oblast	Ivanovo oblast
65a. Aginsk–Buryat AR	Tuva Rep.
27. Voronezh oblast	Yakutia Rep.
28. Kursk oblast	Nenets AR
41. Stavropol Kray	Astrakhan oblast
	Magadan oblast
5th group	Kamchatka oblast
24. Mordovian Rep.	Altai Rep.
46. North Ossetian Rep.	Evenk AR
44. Dagestan Rep.	
45. Kabardino–Balkarsk Rep.	5th group
42. Karachai–Cherkess Rep.	Chukotsk AR
	Koryak AR

Table 2: Continuation.

Classification of living standards of population	Classification of levels of health services and of social–economic development
Kernels Auras	Kernels Auras
Tuva Rep.	Tuva Rep.
1st group	1st group
Kurgan oblast	Kalmyk Rep.
Kemerovo oblast	Kurgan oblast
2nd group	
Altai Territory	2nd group
Orenburg oblast	Aginsk–Buryat AR
Jewish AR	Altai Territory
Kalmyk Rep.	Adigei Rep.
Novosibirsk	Dagestan Rep.
Komi Rep.	Kemerovo oblast
Aginsk–Buryat AR	Komi–Perm AR
Astrakhan oblast	Novosibirsk oblast
Irkutsk oblast	Astrakhan oblast
Perm oblast	Orenburg oblast
Adigei Rep.	Altai Rep.
Primorye Territory	Rostov oblast
Pskov oblast	Chita oblast
Ivanovo oblast	Pskov oblast
Rostov oblast	Ivanovo oblast
Dagestan Rep.	Primorye Territory
Udmurt Rep.	Taimir AR
Komi–Perm AR	
Ust–Ordyn	3rd group
Bryansk oblast	Komi Rep.
Chita oblast	Bryansk oblast
Tomsk oblast	Sakhalin oblast
Nenets AR	Amur oblast
Arkhangelsk oblast	Perm oblast

Table 3:

Kernels Aura	Kernels Auras
Taimir AR	Udmurt Rep.
Sverdlovsk oblast	Buryatiya Rep.
Chelyabinsk oblast	Evenk AR
Tambov oblast	Tambov oblast
Sakhalin oblast	Kaliningrad oblast
Leningrad oblast	Stavropol Territory
Evenk AR	Arkhangelk oblast
Altai Rep.	Ust–Ordyn AR
	Tomsk oblast
3rd group	North Ossetian Rep.
Amur oblast	Irkutsk oblast
Penza oblast	Leningrad oblast
Krasnoyarsk Territory	Krasnoyarsk Territory
Kirov oblast	Chelyabinsk oblast
Nizhni Novgorod oblast	Khabarovsk Territory
Tver oblast	Nenets AR
Kaliningrad oblast	Tver oblast
Krasnodar Territory	Kirov oblast
Buryat Rep.	Sverdlovsk oblast
Stavropol Territiry	
Kostroma oblast	4th group
Khabarovsk Territory	Chuvash Rep.
Samara oblast	Penza oblast
Omsk oblast	Omsk oblast
Vologda oblast	Kabardino–Balkarsk Rep.
Ryazan oblast	Kostroma oblast
Vladimir oblast	Saratov oblast
Chuvash Rep.	Mari Rep.
Saratov oblast	Kamchatka oblast
Mari Rep.	Ryazan oblast
Bashkir Rep.	Magadan oblast
Novgorod oblast	Novgorod oblast
Khakass Rep.	Khakass Rep.
Smolensk oblast	Nizhni Novgorod oblast

Table 3: Continuation.

Kernels	Auras	Kernels	Auras
Karelian Rep.		Samara oblast	
Yaroslavl oblast		Vladimir oblast	
Murmansk oblast		Krasnoyarsk Territory	
Yakutia Rep.		Orel oblast	
North Ossetian Rep.		Karelian Rep.	
Orel oblast		Bashkir Rep.	
Ulyanovsk oblast		Mordovian Rep.	
Kamchatka oblast		Vologda oblast	
		Smolensk oblast	
4th group		Karachai–Cherkess Rep.	
Kaluga oblast		Chukotsk AR	
Lipetsk oblast		Volgograd oblast	
Mordovian Rep.		Yakut Rep.	
Volgograd oblast		Kursk oblast	
Tatar Rep.		Murmansk oblast	
Tula oblast		Lipetsk oblast	
Kursk oblast		Voronezh oblast	
Magadan oblast		Kaluga oblast	
Kabardino–Balkarsk Rep.		Yaroslavl oblast	
Moskow oblast		Koryak AR	
Khanty–Mansi AO		Moscow oblast	
Voronezh oblast		Ukyanovsk oblast	
Karachai–Cherkess Rep.		Tula oblast	
	Yamal–Nenets AR	Tatar Rep.	
Belgorod oblast			Belgorod oblast
5th group		5th group	
Roryak AR		Tyumen oblast	
Tyumen oblast		Khanty–Mansi AR	
Chukotsk AR		St Petersburg	
	St Petersburg	Yamal–Nenets AR	
6th group		6th group	
Moscow		Moscow	

Table 3: Continuation.

non-ferrous metallurgy, the Komy Republic with its pulp and paper industry and mining works, Kemerovskaya oblast' with its chemical industry.

In the second group (very low level), along with the Northern territories, we find the Altai Republic, which shows high sickness rates as it is located close to the Semipalatinsk nuclear proving ground and because this kraj is difficult to access. Irkutskaya oblast' also belongs to this group due to the pulp and paper industrial facilities, Krasnojarskaya oblast' figures here as one of the mining industry centres. St-Peterburg is a large city with highly contaminated environment. Kurganskaya oblast' reveals high infant mortality and alcoholism. Astrakhanskaya oblast' suffers of alcoholism and toxicomania. Primorye Territory (Far East) shows high infant mortality, high morbidity with malignant neoplasms and infectious diseases, short life expectation at birth. Tiumenskaya oblast', Hanty-Mansy and Jamalo-Nenetsky districts, along with all of the typical "Nortern" problems have to face most unfavourable ecological situation owing to their petroleum industry, oil refining facilities, etc. These territories have high infant mortality and high morbidity with the respiration system.

The third group (low level) includes highly industrialized central Russia's areas with large cities. This is where Moscow and Moskovskaya oblast' are to be found. This group incorporates Rostovskaya oblast', an unfortunate exception of the North-Caucasian region which is in general characterized by quite satisfactory parameters, demonstrates high levels of narcotism and alcoholism.

The fourth group (lowered level) unites regions with good climatic and other natural conditions, traditionally rural, such as Central Cnernozemie, Krasnodarsky and Stavropolsky kraj, or with light industry, for instance, Ivanovskaya oblast' and Kostromskaya oblast'. These regions show good life–time parameters. This group also involves the Republics of Volga–Vyatka region — Mary El, Mordovia and Chuvashia with their traditionally high content of young ages, accounted for by some national peculiarities,low morbidity with malignant neoplasms and generally good life–time parameters.

The fifth group (relatively good level) comprises, first of all, the Republics of Northern Caucasus with their low contamination of envi-

Fig. 95: Classification of levels of public health care and environmental conditions from the viewpoint of its influence on the population health.

ronment, clean mountain-air, beautiful climate, namely the Republics Northern Osetia, Dagestan, Kabardino–Balkaria and Karachayevo–Cherkesia. Mordovia, with its satisfactory sickness rates, low alcoholism and narcotism parameters as well as relatively long life expectation at birth, has left the rest of the Republics of Volga-Vyatka region a good deal behind and is also here, in this group.

Note, that the fourth and the fifth groups are mostly formed by the South–Eastern territories of European Russia which are known to have happy natural conditions and fairly low urbanization. There is an apparent tendency for ethnic formations to show better health conditions of the population. This strongly suggest a conclusion that national traditional ways of living make a very important factor affecting the territorial development.

Fig. 95 demonstrates the level of public health care and environmental conditions in Russia, that is the conditions responsible for good health of people.

The first group (deep crisis) is formed of areas with highly contaminated environment: Leningradskaya oblast', Hanty-Mansy AD, Sverdlovskaya oblast', Kemerovskaya, Rostovskaya oblast' (bad water) and Dagestan with its poor development of public health care and social maintenance. But despite this unfavourable state of affairs in this Northern Caucasian Republic, health conditions of its population allow to regard this area as one of the safest.

The second group (very low level) embraces highly contaminated industrialized territories (Moskovskaya, Samarskaya, Cheliabinskaya, Irkutskaya oblast', etc.), Tatarstan, Bashkortostan, Jamalo-Nenetsky autonomous district (pollution due to oil refining and oil-extracting facilities), rural Volgogradskaya oblast' which is undeveloped region with rather contaminated waters and polluted air. In one row with these, there are such a far-away rural area as the Republic of Altai Highlands, Penzenskaya oblast' with its insufficient number of hospitals and polyclinics for children, underdeveloped Karachayevo-Cherkesia which is, like Dagestan, in the preceding classification has won place in the most favourable group. Generally speaking, all the rural areas (Krasnodarsky and Stavropolsky kraj, for one) are usually characterized by underdeveloped social infrastructure and poor medical care.

The third group (low level) includes on the whole central and central-black-soil regions.

The fourth group (lowered level) includes many Northern areas with high proportion of urban population, the so-called "Northern extra-wages" (Magadanskaya, Tiumenskaya oblast', Taimyrsky, Nenetsky, Evenkijsky AD and others). There are also ecologically very clean areas such as Kamchatskaya and Sakhalinskaya oblast'. Among the Southern regions, Kabardino–Balkaria appears to have clean air and water, quite satisfactory level of public health care, and this allows us to place the Republic in our fourth group. Belonging to this group are also Novgorodskaya and Amurskaya oblast', due to their developed social infrastructure and low level of contamination of environment. Moscow and St–Petersburg are also here as being large cities with qualitatively unique level of public health care. But still, aggravated ecological situation does not allow us to place these two centres in the best group.

The fifth group involves two areas with the most favourable combination of all indices, they are Chukotsky and Koryaksky AD which are characterized by high concentration of population in urban areas with developed social infrastructure and safe ecology.

¿From the comparison of the two maps one can see that practically all "leaders" of the first map appear to be among the worst on the second and vice versa. This makes one conclude that better social maintenance cannot be regarded as a guarantee of better health. This observation confirms once again the fact that as few as 10% health of people is accounted for by public health care development. Thus, the study recommends economists and financiers to rely on the first typology, that is the condition of health. So, the recommendation is that in estimating budgetary needs of a particular region, it is our first typology with its orientation on the condition of health, that should be relied upon.

Fig. 96: Classification of living standards of population.

Fig. 96 presents the living standards of Russia's population. The first group characterized by the lowest values includes poor territories of the Kurganskaya oblast', contaminated Kemerovskaya oblast' known for its coal–extracting, metallurgic and chemical industries and unsatisfactory social infrastructure and a far-away and underdeveloped Tuva.

The second group (very low living standards), involves "critical" rural areas with high unemployment (Ivanovskaya, Pskovskaya oblast', Dagestan) and "critical" industrial regions which production was intended for military proposes (Sverdlovskaya, Novosibirskaya, Cheliabinskaya, Rostovskaya oblast' and Udmurtia).

The fourth group (lowered living standards) involves areas rich in natural resources, with high incomes (Hanty-Mansy and Jamalo-Nenetsky autonomous districts); in old traditionally rural black earth areas with highly developed agriculture and conservative government (Belgorodskaya, Lipetskaya, Kurskaya oblast'); territories showing good health and high incomes (Karachayevo-Cherkesia - the Soviet phenomenon with high "shadow" incomes of population - reveals highest per capita values of autotransport available); areas exhibiting high incomes due to "the Northern" extra-wages (Magadanskaya oblast'). This group also includes Kaluzhskaya oblast' which happens to show low unemployment, low percent of poor population, satisfactory criminal situation, good personal autotransport supply, high percent of specialists with higher education.

The medium standards of living (the fifth group) is shown by the oil–producing Tuimenskaya oblast' with high incomes, the Korjaksky and Chukotsky AD with small population concentrated in the urban areas and few crimes, good ecology and increased incomes. These districts show similarly high rating because they possess relatively developed social infrastructure. Although the fact of having a developed infrastructure alone does not necessarily imply high standards of living, in the case under discussion it is this very index that has appeared

Fig. 97: Classification of levels of social–economic development.

determining in deciding the position of these areas. The thing is that provided high levels of urbanisation and dispersed population, all of the social infrastructure

is concentrated in towns. St–Petersburg, due to its highly developed social infrastructure, special "postindustrial" situation, also belongs to this successful group. It is easy to guess that the highest standards of living are, naturally, exhibited by Moscow which has got the same pre-requisites as has St-Petersburg, but moreover, Moscow being the capital, shows high incomes, lowest unemployment and is an indisputable centre in every respect.

Fig. 97 is a map revealing social and economic development of Russia's regions. A privileged, special, position is, again, occupied by Moscow successful social perquisites are strengthened by economic ones. Along with that, Moscow is the greatest economic inter-mediate.

The first 5 positions are occupied by Moscow, St-Petersburg and the oil–rich territories of the Tuimenskaya oblast', Jamalo–Nenetsky and Hanty–Mansy autonomous districts belonging to the Tiumenskaya oblast' (relatively good and medium level). The latter four form the fifth group.

The lowered development (the fourth group) is found in the Belgorodskaya oblast', Tatarstan, Uljanovskaya oblast' but it should be mentioned that such a favourable state of affairs here is somewhat artificial and is mostly due to the subsidiary state policy with respect to these territories rather than due to any other reasons. Moskovskaya oblast' (one of the economically strongest regions) is also in this group. Some northern areas having rich natural resources and large percent of urban population (Murmanskaya, Krasnojarskaya oblast', Chukotsky AD, Jakutia, Karelia, Hakasia) and some black earth rural areas with relatively highly developed

agriculture and metallurgy (the Central Chernozemye) have come into this group as well.

Such regions as Kaluzhskaya oblast', have shown so successfully because of particularly low decline in agricultural production (this oblast' is one of the seven "best" regions) along with high profits nearly in every branch of production. The Vologodskaya oblast' has high rating in many fields; lowest decline in industrial and agricultural production, low unprofitableness of a region's economy and sufficiently high investing attractiveness and sufficiently high export index, the exception being enterprises using foreign investments. Other representatives of this group are the Penzenskaya oblast' showing well in unprofitableness, investing attractiveness and export index; the Kostromskaya oblast' (low agricultural production decline, low rating of unprofitableness), Mary El (big volumes of production at enterprises applying foreign capital and low decline in agricultural production); the Ryazanskaya oblast' (low unprofitableness, good profits in all brunches of production, active participation of foreign investors in production).

The level of deep social-economic crisis is exhibited in the underdeveloped Kalmykia and the Hebrew autonomous Republic, in the rural and outlying areas of the Kurganskaya oblast' with its high unemployment. Very low level of social-economic development is found in the difficult–to–access and underdeveloped Altai with its ecological problems, in the "critical" areas with high unemployment (Pskovskaya, Ivanovskaya oblast'), and some others.

Now let us compare the two classifications discussed above — that in terms of health condition of the population and the one with respect to environmental health — with the classification of Russia's regions referring to per capita budgetary expenditures on public health care. It is easy to notice apparent correlation thereof.

Thus, the per capita budgetary expenditures on public health care cannot be taken alone as the criterium of efficiency of the public health care system in the country. The present structure of budgetary expenditures on public health care is such that it produces little any effect on the system on the whole and have to be revised significantly.

To sum up the research presented allows to estimate the following four aspects of social-economic situation in the component units of the Russian Federation:

— health condition of the population;

— public health care and environmental health;

Classification of levels of per head budged expenses for health services		Classification of levels of health services and of the state of the environement from the point of view of its influence on the population health	
Kernels	Auras	Kernels	Auras
1st group			
Chuvash Rep.			Leningrad oblast
Dagestan Rep.			Khanty–Mansi AR
Penza oblast			
Rostov oblast		1st group	
Volgograd oblast		Sverdlovsk oblast	
Kursk oblast		Rostov oblast	
Orel oblast		Kemerovo oblast	
Mari Rep.		Dagestan Rep.	
Novgorod oblast			
Kaluga oblast		2nd group	
Kurgan oblast		Orenburg oblast	
Buryat Rep.		Krasnodar Territory	
Bryansk oblast		Chelyabinsk oblast	
Tula oblast		Stavropol Territory	
Mordovian Rep.		Murmansk oblast	
Tambov oblast		Samara oblast	
Pskov oblast		Irkutsk oblast	
Stavropol oblast			Vologda oblast
Ivanovo oblast			Bashkir Rep.
Smolensk oblast			Yamal–Nenets AR
Karachai–Cherkess Rep.			Tatar Rep.
Saratov oblast			Moscow oblast
			Komi Rep.
2nd group			Nizhni Novgorod oblast
North Ossetia Rep.			Perm oblast
Astrakhan oblast			Penza oblast
Leningrad oblast			Karachai–Cherkess Rep.
Kabardino–Balkarsk Rep.			

Table 4:

Kernels Auras	Kernels Auras
Voronezh oblast	3rd group
Udmurt Rep.	Kostroma oblast
Kirov oblast	Tver' oblast
Aginsk-Buryat AR	Kaluga oblast
Lipetsk oblast	Lipetsk oblast
Tver' oblast	Tula oblast
Khakass Rep.	Buryat Rep.
Altai Territory	Tyumen' oblast
Kaliningrad oblast	Ulyanovsk oblast
Belgorod oblast	Primorye Territory
Tomsk oblast	Kurgan oblast
Ust–Ordyn AR	Novosibirsk oblast
Ryazan oblast	Bryansk oblast
Kostroma oblast	Krasnoyarsk Territory
Ulyanovsk oblast	Saratov oblast
Vladimir oblast	Ust–Ordyn AR
Yaroslavl oblast	Altai Territory
Krasnodar oblast	Tambov oblast
Orenburg oblast	Voronezh oblast
Novosibirsk oblast	Orel oblast
	Adigei Rep.
3rd group	Volgograd oblast
Chelyabinsk oblast	Ryazan oblast
Tatar Rep.	Belgorod oblast
Bashkir Rep.	Arkhangelsk oblast
Moscow oblast	Omsk oblast
Omsk oblast	Kursk oblast
Nizhni Novgorod oblast	Smolensk oblast
Jewish AR	Udmurt Rep.
Chita oblast	Pskov oblast
Perm oblast	Chuvash Rep.
Adigei Rep.	Kaliningrad oblast
Kalmyk Rep.	Komi-Perm AR
St Petersburg	

Table 4: Continuation.

Kernels	Auras	Kernels	Auras
Vologda oblast		4th group	
Irkutsk oblast		Vladimir oblast	
Karelian Rep.		Yaroslavl oblast	
Primorye Territory		Tomsk oblast	
Sverdlovsk oblast		Chita oblast	
Amur oblast		Mordovian Rep.	
Komi–Perm AR		Taimir AR	
Arkhangelsk oblast		Kabardino–Balkarsk Rep.	
Krasnoyarsk Territory		Agin–Buryat AR	
Samara oblast		Mari Rep.	
Khabarovsk Territory		Kirov oblast	
Tyumen oblast		Jewish AR	
Kemerovo oblast		Khabarovsk Territory	
Komi Rep.		Karelian Rep.	
Altai Territory		Nort Ossetia Rep.	
		Kalmyk Rep.	
4th group		Sakhalin oblast	
Sakhalin oblast		Novgorod oblast	
Nenets AR		St Petersburg	
Murmansk		Khakass Rep.	
Kamchatka oblast		Amur oblast	
Tuva Rep.		Moscow	
Yamal–Nenets AR		Ivanovo oblast	
Moscow		Tuva Republic	
Yakutia Rep.			Yakutia Rep.
Khanty–Mansi AR			Nenets AR
Magadan oblast			Astrakhan oblast
Taimir AR			Magadan oblast
Evenk AR			Kamchatka oblast
			Altai Rep.
5th group			Evenk AR
Chukotsk AR			
Koryak AR		5th group	
		Chukotsk AR	
			Koryak AR

Table 4: Continuation.

— standards of living;
— social-economic development.
Assuming that the resulting classifications are based on indices having impact on budgetary expenditures, it has become possible to achieve objective estimates of the real budgetary needs of this or that territory.

Geographic analysis of the current situation presented on the anamorphosises shows explicitly all the advantages and disadvantages of each group of regions and can be applied in the reforming of the budgetary system (for instance, budgetary policy in the domain of public health care). Moreover, the criteria proposed and the classifications on account thereof can well be used to find out particular regions actually in need of subsidies from, say, the fund of region's development.

Just the geographical analysis which takes into account interregion differences can make an essential contribution to elaboration of a new up–to–date system of financial relations between the Federation and its component units.

Conclusion

Completing the book let us fix most essential reasons for practical use and perspective of anamorphosises. First of all, and this is not the most principal, they are convincing as illustrations, which permit to imagine visually some non-evident facts and even to see some hidden geographical regularities. For example, theoretical principles of V.Cristaller and A.Lösch about regularities of arrangements of hierarchical systems of centres get their confirmation only in regions with uniform distribution of population. It is possible that search of regularities in arrangements of populated places in the plane with the uniformed phenomenon could make the Cristaller–Lösch principle much more often observable or even universal. It has good prospects to use anamorphosises for optimization of arrangement of nets of education, medical facilities and other service centres which, in general should be distributed uniformly on the artificially uniformed demographic plane.

In the second place, anamorphosises make the relations between phenomena more visual if they are analysed on the background of an image based on characteristics connected with them. Thirdly, it is reasonable to use anamorphosises for prediction of development of diffusive processes which take place in a non-uniform environment If one transforms the resistance to the development of the diffusion into the uniform one, then most probably the diffusion will develop concentrically from the initial point. Thanks to that it is possible to forecast its development in time and to represent it in a graphical form. To get the picture of the spreading of the diffusion in the non-uniform environment, one should restore the image into the initial form. Anamorphosises can be used for the study of the diffusion of pollutions in the atmosphere and hydrosphere and also for a number of other problems. The same one can say about compilation of maps of transport accessibility, which are created on the background of uniform practicability.

Thus compilation of anamorphated images in a number of cases is reasonable for modelling the structure, interconnections and dynamics of geographical phenomena. We hope that anamorphated images will attract attention of geographers of different interests and will become not eccentric illustrations, but tools of real geographical analysis. This field can appear to be a non-upturned scientific virgin land, which will give a rich harvest after a skilful processing.

Literature

Angel S, Hyman G. Urban Fields, 1976, Pion, London, 178 p.

Arkhipov Yu.R., Blazhko N.I., Grigor'ev S.B., Zabotin Ya.I. Trofimov A.M., Khuzeev R.G. Mathematical methods in geography. Kazan. Kazan University Publisher, 1976, 352 pp (in Russian).

Armenakis, C., Siekierska, E.M. Issues on the Visualization of Time-Dependent Geographical Information. Proceedings of the Canadian Conference on GIS, 1991, p.584–592.

Atkinson G., Hamilton K., Accounting for progress: indicators for sustainable development. Environment. Wash., 1996, Vol. 38, N 4, p. 16–20, p.40–44.

Atlantic ocean. Kort V.G., Sal'nikov S.S. (Edts), Leningrad, Nauka Publisher, 1984, 589 pp (in Russian).

Belov A.L. Political geography of Canada: main contemporary problems. — All-Union Geographical Society Izvestiya, 1983, v.115, no.4, p.363–368 (in Russian).

Bestuzhev-Lada I.V. Extinction of Russia's population: myth or reality? Otechestvennaya istoriya, 1996, N 4, p.113–128 (in Russian).

Bobylev S.N. Ecologization of economy and the business. Ekologiya. Ekonomika. Biznes. (Ekologo-ekonomicheskiye aspekty ustoichivogo razvitiya) Moscow: IRIS-PRESS Pablishers, 1995, p.134–142 (in Russian).

Bochkareva T.V. Tipology of largest urban aglomerations of the USA by the quality of environment and principal factors of its formimg. — Academy of Sciences of the USSR, Izvestiya, ser. geogr., 1981, N 5, p.74–85 (in Russian).

Bochkareva T.V. Influence of the ecological factor on changes of the dynamics and of the distribution of population in largest urban aglomerations of the USA. — All–Union Geographical Society, Izvestiya, 1983, v. 115, N 3, p.259–266 (in Russian).

Borden D.D. A note of importance of shape in cartogramm communication. — The Journal of Geography, 1976, Vol. 3, N 1, p.21–31.

Borden D.D. Communication aspects of value–by–area cartograms. -– The American Cartographer, 1975, Vol. 2, N 4, p.154–168.

Bryukhanov A.V., Tikunov V.S. Photographic method of compilation of simple anamorphated images. — Geodesy, cartography, and air photography, Issue. Lvov, "Vishcha shkola" Publisher, 1983, p.117–122 (in Russian).
Bugaevskiy Yu.L. Varivalent projection of the pseudocylindric type for anamorphated maps. — Izvestiya vuzov. Geodesy and air photography, 1987, N 1, p.100–107 (in Russian).
Bulatov R.P, Barash M.S., Ivanenkov V.N., Marti Yu.Yu. Atlantic ocean. Moscow, 1977, 296 pp (in Russian).
Bunge W. Theoretical Geography. Lund Studies in Geography, Series C, Number 2. Lund: G.W.Gleerup, 1966.
Care of the Earth. The strategy of sustainable development IUCN/UNEP/WWF, 1991, 34 pp (in Russian).
Cerny J. Ontogenetic and Phylogenetic Perspectives on Cartograms. — The Monadnock, 1972, 46, p.47–52.
Concept of the Russian Federation's transition to sustainable development. Zeleny mir, 1996, N 2 (in Russian).
Cuenin R. Cartographie genérale. Tome 1. Notions générales et principes d'élaboration. Paris, 1972, p.309–310.
Dahl A.L. Measuring the unmeasurable. Our planet/UNEP, 1996.
Dal V. Explanatory dictionary of the living Great Russian language. V. 2. St Petersburg – Moscow, 1881, 779 pp (in Russian).
Davydov A.A. Index of social trouble. Socis, 1995, N 10, p.118– 128 (in Russian).
Dent B. A Note on the Importance of Shape in Cartogram Communication. — Journal of Geography, 1972, 71, p.393–401.
Dent B.D. Communication aspects of value–by–area cartograms. — The American Cartographer, 1975, v. 2, N 2, p.154–168.
Dorling D. Map design for census mapping. — The Cartographic Journal, 1993, v. 30, N 2, p.167–183.
Dorling D. From computer cartography to spatial vizualisation: a new cartpgram algorithm — The Proceedings of Auto–Carto 11, 30 October–1 November 1993, Minneapolis, Minnesota, 1993, p.208–217.
Dougenik J.A., Chrisman N.R., Niemeyer D.R. An algorithm to construct continuous area cartograms. — Prof. Geogr., 1985, 37, N 1, p.75–81.

Dutton, G.H. American graph fleeting, a computer–holograph map animation of United States population growth 1790-1970. Computer mapping in education research and medicine. Harvard University. Laboratory for computer graphics and spatial analysis, 1979, p.53–62.

Eastman Ir., Nelson W., Shields G. Production consideration in isodensity mapping. — Cartographica, 1981, Vol. 18, N 1, p.24–30.

Economic geography of the World ocean. Leningrad, Nauka Publisher, 1979, 311 pp (in Russian).

Elsasser H. Nichtflächenproportionale Kartogrammartige Darstellungen der Schweiz. — Geogr. helv., 1970, 25, N 2, s.78–82.

Ewing G. Multimensional scaling and time–space maps. — The Canadian Geographer, 1974, V. 18, N 2, p.161–167.

Ewing G.O., Wolfe R. Surface feature interpolation on two–dimen sional time–space maps. — Environment and planning, 1977, v. 9, N 4, p.429–437.

Fedotov A. The Earth planet, the mankind and the economy. Ekonomist, 1995, N 11, p.43–56 (in Russian).

Forster F. Use a demographic base map for the presentation of areal data in epidemiology. — British Journal of Preventive and Social Medicine, 1966, Vol. 20, N 4, p.165–171.

Getis A. The determination of the location of retail activities with the use of a map transformation. — Economic Geography, 1963, N 39, p.14–22.

Gornostaeva G.A., Tikunov V.S. Contemporary features of the economic structure of cities of the Moscow oblast. — Moscow State University, Vestnik, ser. geogr., 1988, N 3, p.20–29 (in Russian).

Gornostaeva G.A., Tikunov V.S. Mathematical–Cartographical methods of study of dynamics of functions of cities of the Moscow region. — In: Problems of the complex mapping of the Moscow region. Moscow, Moscow branch of the All–Union Geographical Society, 1988, p.33–49 (in Russian).

Gornostayeva G.A., Tikunov V.S. Current Characteristics of the Economic Structure of Moscow Oblast Cities. — Soviet Geography: Rev. and Transl., 1989, XXX, N 2, p.91–102.

Gorshkov V.G., Kondratiev K.Ya., Danilov-Daniliyan V.I., Losev K.S.

Okruzhayushchaya sreda: ot novykh telhnologiy k novomu myshleniyu (Environment: from new technologies to new thinking). Moscow, 1994 (in Russian).
Gorshkov V.G., Kondratiev K.Ya., Losev K.S. If the wisdom of nature is leaned for support. Vesniik RAN, 1996, V. 66, N 2, p.119–128 (in Russian).
Gould P., White R. Mental Maps, 1974, Penguin Books Inc., New York, Baltimore, 204 pp.
Griffin T.L. Cartographic transformation of the thematic map base. — Cartography (Austral.), 1980, 11, N 3, p.163–174.
Griffin T.L.C. Recognition of areal units on topological cartograms. — Amer. Cartogr., 1983, 10, N 1, p.17–29.
Gusein-Zade S.M. Bunge's problem in central place theory and its generalizations. — Geographical Analysis, 1982, 14, N 3, p.246–252.
Gusein-Zade S.M., Suetova I.A., Tikunov V.S. Experience of correlation analysis of phenomena with the use of anamorphated images. — Geodesy and cartography, 1993, N 12, p.40–45 (in Russian).
Gusein-Zade S.M., Tikunov V.S. Numerical methods of compilation of anamorphated cartographic images. – Geodesy and cartography, 1990, N 1, p.38–44 (in Russian).
Gusein-Zade S.M., Tikunov V.S. A method of compilation of anamorphated cartographic images. — In: Ecological mapping on the contemporary stage. Book I, Leningrad, 1991, p.29–31 (in Russian).
Gusein-Zade S.M., Tikunov V.S. Experience of compilation and use of anamorphosises. — Izvestiya vuzov. Geodesy and air photography, 1991, N 4, p.142–152 (in Russian).
Gusein-Zade S.M. Construction of anamorphated images for geographic studies. — Moscow State University, Vestnik, ser. geogr., 1992, N 4, p.43–52 (in Russian).
Gusein-Zade S.M., Tikunov V.S. New methods of compilation of anamorphosises and their using. — Proceedings 5th International Symposium on Spatial Data Handling. August 3–7, 1992, Charleston, South Carolina, USA, v.1, p.51–62.
Gusein-Zade S.M., Tikunov V.S. Arrangements of Systems of Central Places and their Representation with the Use of Anamorphosises. — European Regional Science Association. The 33rd European Congress.

Moscow, Russian Federation, August 24–27, 1993, Abstracts of papers, 1993, p.68.
Gusein-Zade S.M., Tikunov V.S. A New Technique for Constructing Continuous Cartograms. — Cartography and Geographic Information Sys- tems, 1993, v.20, N 3, p.167–173.
Gusein-Zade S.M., Tikunov V.S. The Transformed Image in Geographical Analysis. — Mapping Sciences and Remote Sensing, 1993, v. 30, N 4, p.306–319.
Gusein-Zade S.M., Tikunov V.S. Compilation of Linear Transformed Images. — Mapping Sciences and Remote Sensing, 1994, v. 31, N 1, p.33–48.
Gusein-Zade S.M., Tikunov V.S. Analog Methods in the Compilation of Areal Transformed Images. — Mapping Sciences and Remote Sensing, 1994,
v. 31, N 1, p.49–65
Gusein-Zade S.M., Tikunov V.S. Numerical Methods in the Compilation of Transformed Images. — Mapping Sciences and Remote Sensing, 1994, v. 31, N 1, p.66–85
Gusein-Zade S.M., Tikunov V.S. Applications of Transformed Images in Geographical Analysis. — Mapping Sciences and Remote Sensing, 1994, v. 31, N 2, p.107–123
Gusein-Zade S.M., Tikunov V.S. The use of anamorphosises for a
description of zones of the influence of big urban centers. — Abstracts of Papers and Posters, presented at the 34–th European Congress of the Regional Science Association. Groningen, The Netherlands, 23–26 August 1994, p.114.
Gusein-Zade S.M., Tikunov V.S. The use of anamorphosises for a description of zones of the influence of big urban centers. — 34–th European Congress of the Regional Science Association. Groningen, The Netherlands, 23–26 August 1994, p.1–18.
Hägerstrand T. Migration and area: survey of a sample of Swedish migration fields and hypothetical considerations on their genesis. — Lund Studies in Geography, Ser. B, Human Geography, 1957, V. 13, p.27–158.
Härö A. Area Cartogram of the SMSA Population of the United States. — Annals of the Association of American Geographers, 1968,

V. 58, N 3, p.452–460.
Harris C., McDowell G. Distorted Maps — A Teaching Device. — Journal of Geography, 1955, 55, p.286–289.
Harman H.H. Modern factor analysis. Moscow, Mir Publisher, 1982, 486 pp (in Russian).
Hua Yi–xin Making and application of value–by area topological cartogram. — 14th World Conf. Int. Cartogr. Assoc., Budapest, 17–24 Aug., 1989: Abstr., Budapest, 1989, p.600.
Hunter J.M., Meade M.S. Population modeles in the high school. — The Journal of Geography, 1971, Vol.70, N 2, p.95–104.
Hunter J.M., Young J.C. Technique for the Construction of quantitative Cartograms by Physical Accretion Models. — The Professional Geographer, v.XX, N 6, 1968, p.402–407.
Ewing G., Wolfe R. Interpolation of properties of a surface on two-dimensional maps with the space as a function in time — Cartography. Issue 1. Foreign conceptions and directions of investigations. Moscow, "Progress" Publisher, 1983, p.161–173 (in Russian).
Jackson C.J. How to make a population map. — Canadian Geographical Journal, 1974, Vol. 89, N 5, p.26–29.
Kadmon N. Data–bank derived hyperbolic–scale equitemporal town maps. — International Yearbook of Cartography, 1975, v.15, p.47–54.
Kadmon N. Cartograms and topology. — Cartographica, 1982, 19, N 3–4, p.1–17.
Kadmon N. Photographic, polyfocal and polar–diagrammatic mapping of atmospheric pollution. — Cartogr. J., 1983, V. 20, N 2, p.121–126.
Kadmon N., Shlomi E. A polifocal projection for statistical surfaces. — The Cartographic Journal, 1978, Vol. 15, N 1, p.36–41.
Kartogramm zür Reichstagswahl. 2 Wahlkarten d. Deutschen Reiches in alter und neuer Darstellung mit polit.–statist. Begleitworten und kartogr. Erl. von Dr. H.Haack und H.Wiechel. Gotha: J.Perthes, 1903, 36 s.
Kelly J.I., Neville R.J.W. A population cartogram of New Zealand. — N. Z. J. Geogr., 1985, N 79, p.7–11.
Khilchevskaya R.I. Problems of environmental economy in the context of sustainable development concept. Ekon. i mat. metody, 1996, V.32,

N 3, p.85–95 (in Russian).
Kolosov V.A., Tikunov V.S., Zayats D.V. World in the mirror of mass media: use of anamorphosises in the political-geographical analysis. — Moscow State University, Vestnik, ser. geogr., 2000, N 2, p.3–7 (in Russian).
Kvasha A.Ya. Demographic and economic development of Russia. Vestnik Mosk. Un-ta. Ser. ekon., 1995, N 5, p.75–80 (in Russian).
Lacko L. The Form and the Contents of Economic Maps. — Tijdschrift roor Econ. en Soc. Geografie, 1967, p.324–330.
Levison M., Haddon W. The area adjusted map: an epidemiologilal device. — Public Health Reports, 1965, v.80, N 1, p.55–59.
Losev K.S., Gorshkov V.G., Kondratev K.Ya., Kotlyakov V.M., Zalikhanov M.Ch., Danilov-Danilyan V.I., Gavrilov I.T., Golubev G.N., Revyakin V.S., Grakovich V.F. Ecological problems of Russia, Moscow, 1993, 348 pp (in Russian).
Malinovsky D.V., Tikunov V.S., Trejvich A.I. Cartographic evaluation of changing mutual transport remoteness between Russian regions in 1985-2001 (at the example of railway tariff distances) - Proceedings of the International Conference "GIS for Sustainable Development of Territories. InterCarto 8, Helsinki, St.Petersburg, 2002, p.180-186 (in Russian).
Malkhazova S.M., Phillips D.R., Tikunov V.S., Verhasselt Y. Public Health and Environmental Pollution in Russia: A Methodology for the Assessment of Critical Ecological Situations. — Bulletin of International Geographical Union, 1997, v. 47, N 1, p.5–16.
Malkhazova S.M., Tikunov V.S. Use of anamorphosises for study of space differences in distribution of natural–endemic diseases. — Russian Geographical Society, Izvestiya, 1993, v.125, N 5, p.86–93 (in Russian).
Malkhazova S.M., Tikunov V.S. Medical–geographical approach to the valuation of crisis ecological situations. — In: Geography, Moscow State University Publisher, 1993, p.171–181 (in Russian).
Malkhazova S.M., Tikunov V.S. Analysis of Spatial Differences in the Health of the Population of the World (with the Examples of Natural–Endemic Diseases). — InterCarto: GIS for Environmental Studies and Mapping Conference. Proceedings. 23–25 May 1994, Moscow, Russia,

1994, p.105–110.
Meadows D.X., Meadows D.L., Randers I. Za predelami rosta (Beyond the limits to growth) Moscow, Moscow State University, 1994 (in Russian).
Mitchel G. Problems and fundamentals of sustainable development indicators. Sustainable Development, 1996, V.4, N 1.
Molochko, A.N. Animation principles in cartograhic modelling. In: Kartograficheskoye obespecheniye osnovnykh napravleniy ekonomicheskogo i sotsyalnogo razvitiya USSR i eyo regionov. Tez.dokl. 6 Resp. nauchn. konf. Chernovtsy, 1987, p.214–215 (in Russian).
Monitoring environmental progress (a report on work in progress). Washington, DC: World Bank, 1995, 72 pp.
Monkhouse F.J., Wilkinson H.K. Topological Maps. Book: Maps and diagrams. Their compilation and construction. London, 1971, 85 pp.
Muller J.–C. The mapping of travel time in Edmonton (Alberta). — Can. geogr., 1978, v.22, N 3, p.195–211.
Muller J.–C. La cartographie d'une métrique non euclidienne: les distances–temps. — L'Espace Géographique, 1979, v.8, N 3, p.215–227.
Muller J.–C. Euclidean geographic spaces: mapping functional distances. — Geogr. Anal., 1982, v.14, N 3, p.189–203.
Muller J.–C. La cartographie des espaces fonctionnels. — L'Espace géogr., 1983, v.12, N 2, p.142–152.
Muller J.–C., Honsaker J.L. Non–Euclidean maps: an improved procedure to portray travel times. — Albertan Geogr., 1980, N 16, p.27–40.
Murdych Z. K otázce středních míst a kartografických metod jejich vyjadřování. — Acta Universitatis Carolinae. Geographica, 1969, N 2, s.21–32. (in Czech).
Murdych Z. Metody anamorfózy městských plánů. — Acta Universitatis Carolinae. Geographica, 1971, N 1–2, s.111–118. (in Czech).
Murdych Z. On the subject and methods of map anamorphosis. — Cartography. Issue 1. Foreign conceptions and directions of investigations. Moscow, "Progress" Publisher, 1983, p.152–160 (in Russian).
Myagkov S.M. Social environmental stability of territorial economic complexes. Vestnik MGU, Ser. Geograf., 1996, N 2, p.3–11 (in Russian).

Myagkov S.M., Tikunov V.S. Transition to sustainable development: the geographical foundation. Vestnik MGU. Ser. Geograf., 1998, N 4, p.71–80 (in Russian).
Olekh L.G. Challenges of the social philosophy. Gumanitarnyye nauki v Sibiri, 1996, N 1, p.3–8 (in Russian).
Olson J. Noncontiguous Area Cartograms. — The Professional Geographer, 1976, 28, N 4, p.371–380.
Ormeling, F. Ariadne's thread - structure in multimedia atlases. Proceedings of the 16th International Cartographic Conference. Cologne, 3-9 May 1993, vol. 2, Bielefeld, 1993, p.1093–1100.
Papenov K.V. Sustainable development: theoretical aspects. Vestnik MGU, Ser. Ekon., 1995, N 5, p.3–18 (in Russian).
Pavlova, Z.G., Korneev, A.A., Chalov, V.P. Holographic methods used for the problems of forest typology. In: Aerokosmicheskiye metody issledovaniya lesov. Tez. dokl. Vses. konf., Krasnoyarsk, 7-9 iyulya 1984 g., Krasnoyarsk, 1984, p.171–172 (in Russian).
Perelet R.A. International aspects of the environmental business and the transition to sustainable development. Ekologiya. Ekonomika. Biznes. (Ekologo-ekonomicheskiye aspekty ustoichivogo razvitiya) Moscow: IRIS-PRESS Publishers, 1995, p.152–166 (In Russian).
Perspective Canada: a compendium of social statistics. Ottawa, 1977, v.2, 335pp.
Petrov P.V., Serbenyuk S.N., Tikunov V.S. An analytic method of compilation of anamorphated cartographic images. — Moscow State University, Vestnik, ser. geogr., 1983, N 2, p.56–63 (in Russian).
Petrov P.V., Serbenyuk S.N., Tikunov V.S. An analytic technique of map transformation. — Mapping sciences and remote sensing, 1984, 21, N 1, p.49–57.
Petrov P.V., Tikunov V.S. Modelling of isochrons for compilation of maps of transport accessibility. — Geodesy and cartography, 1984, N 5, p.42–43 (in Russian).
Philips' New World Atlas. London, George Philip, 1976.
Pipkin J.S., Tikunov V.S. Use of anamorphosises for study of social–economic objects. — All–Union Geographical Society Izvestiya, 1989, v. 121, N 5, p.433–437 (in Russian).
Pogrebinskaya V. Living standards of the population and the income

structure. Ekonomist, 1995, N 10, p.18–26 (in Russian).
Poponin V.I. The use of the low of light refraction for calculation of isochrons on maps of accessibility. — Moscow State University, Vestnik, Ser.5, Geography, 1972, N 5, p.95–98 (in Russian).
Pravda J. A torzított kartogram. — Geod es kartogr., 1977, v.29, N 4, o.274–279. (in Hungarian).
Pravda J. Zakon kartogramu a problem vyjadrovania nerozlohových charakteristík. — Geografický časopis, 1983, 35, N 2, s.136–159. (in Slovak).
Pravda J. Metodicko–vyjadrovacie problémy tvorby tematických map. Bratislava, 1983, 64 s. (in Slovak).
Prioritety dlya partnerstva. Alternativny plan deistvii po okruzhayushchei srede. (Priorities for partnership. Alternative action plan for the environment) Moscow: EKOPRESS-3M, 1996, 88 pp (in Russian).
Raisz E. The rectangular statistical cartogram. — Geographical Review, 1934, Vol.24, N 2, p.292–296.
Raisz E. Rectangular Statistical Cartograms of the World. — Journal of Geography, 1936, 35, N 1, p.8–10.
Raisz E. General cartography, 1948, New York, Toronto, London, 354 pp.
Raykh E.L. Modelling in medical geography. Moscow, Nauka Publishers, 1984, 156 pp (in Russian).
Raykh E.L., Maksimova L.V. Medical–geographical territorial differentiation (on the example of characteristic endemic diseases). — Academy of Sciences of the USSR, Izvestiya, ser. geogr., 1988, N 6, p.34–43 (in Russian).
Ralf B. Multimedia GIS - definition, requirements and applications. The 1994 European GIS Yearbook. p.151–154.
Raspolozhenskiy N.A., Sventek Yu.V., Tikunov V.S. On possibilities of the use of electric modelling in geography. — Moscow State University, Vestnik, ser. geogr., 1972, N 4, p.32–37 (in Russian).
Raspolozhenskiy N.A., Sventek Yu.V., Tikunov V.S. Automation of construction of maps compiled with the use of analog models. — In: Construction of cartographic images with the help of computers and automatic devices. Kalinin, 1973, p.55–66 (in Russian).
Raspolozhenskiy N.A., Sventek Y.V., Tikunov V.S. The use of electric

models in geography. — Soviet Geography: Rev. and Transl.,1973, 14, N 5, p.315–321
Raspołożenski N.A., Swientek J.W., Tikunow W.S. Możliwości zastosowania modelowania elektrycznego w geografii. — Przestrzenne modele symulacyjne. Przegląd zagranicznej literatury geograficznej., 1977, N 2, s.145–153 (in Polish).
Raspolojensky N.A., Sventek Y.V., Tikounov V.S. L'automatisation du procede d'etablissement des images anamorphises. — Rapport présenté à la session de la Comission pour l'automatisation de l'AIC. Budapest, 1973. Moscou, 1973, 10 pp.
Raspolozhenskiy N.A., Sventek Y.V., Tikunov V.S. Automation of compilation process of anamorphotic images. — Automation the new trend in cartography. Budapest. 1974, p.208–213.
Raveneau, J.–L., Miller, M., Brousseau, Y., Dufour, C. (1991) Micro–atlases and the diffusion of geographic information: an experiment with HyperCard. Geographic Information Systems: The micro-computer and modern cartography, Pergamon Press, 1991, v.1, p.201–223.
Rodoman B.B. Polarization of the landscape as means of preservation of the biosphere and recreational resourses. — In: Resourses, Environement, Population. Moscow, "Nauka" Publishers, 1974, p.150–162 (in Russian).
Rodoman B.B. Geographical cartoids. — In: Theory and methods of economic–geographical researches. Moscow: Moscow Branch of the Geographical Society, 1977, p.15–34 (in Russian).
Romanenko L.M. Methods of investigating the Russian society. Socis, 1995, N 1, p.127–131 (in Russian).
Ruston G. Map Transformations of Point Patterns: Central Place Patterns in Areas of Variable Population Density. — Papers and Proceedings, Regional Science Assn., 1971, v.28, p.111–129.
Saalfed A. A fast rubber–sheeting transformation using simplicial coordinates. — The American Cartographer, 1985, V.12, N 2, p.169–173.
Salishchev K.A. Some features of the contemporery development of cartography and their theoretical sense. — Moscow State University, Vestnik, Ser.5, Geography, 1973, N 2, p.3–12 (in Russian).
Salishchev K.A. Cartology. Moscow, Moscow State University Publishers, 1982, 408 pp (in Russian).

Santein K.P. Information, please. Konstituts. pravo: vostochnoevrop. Obozreniye, 1993, N 3(4), p.52–55 (in Russian).
Sattar D. Indicator Abstracts. New Economic Found Report, 1995, 9pp.
Selvin S., Merrill D., Sacks S., Wong L., Bedell L., Schulman J. Transformations of maps to investigate clusters of disease. Lawrence Berkeley Laboratory, Univ. of California, 1984, 33p.
Sen A. On a Class of Map Transformations. — Geographical Analysis, 1976, 8, N 1, p.23–37.
Siekierska, E.M. From the electronic atlas system to the ecectronic atlas products (electronic atlas of Canada from the beginning to the end).- Proceedings of the Seminar on Electronic Atlases, Visegrad, Hungary, April 27-29, 1993, p.103–111.
Sigolaeva E.A., Tikunov V.S. Mathematical-cartographic modelling of economic development of the Atlantic ocean. — Geography and Natural Resourses, 1986, N 4, p.113–121 (in Russian).
Skoda L., Robertson J.C. Isodemographic map of Canada. — Geogr. Pap. Lands Dir. Dep. Environ., 1972, N 50, VIII, 34 p.
Sliwa A. Kartograficzne metody ekonomicznej transformacji przestrzeni. — Folia geogr. Ser. geogr.–econ., 1978, v.11, s.144–160. (in Polish).
SNiP II–60–75, part II, chapter 60. Planning and building of cities settlements and rural populated places. Moscow, Strojizdat Publisher, 1979, 77 pp (in Russian).
Sukhomlinova V.V. Systems of "society" and "nature": diversity, stability, evolution. Obshchestv. nauki i sovremennost', 1994, N 4, p.135–141 (in Russian).
Suvorov A.K. Topology and transformation of cartographic images. — Geodesy and Cartography, 1984, N 12, p.28–31 (in Russian).
Suvorov A.K. On the relation between traditional and nontraditional space images. — Academy of Sciences of the USSR, Izvestiya, ser. geogr., 1984, N 5, p.121–133 (in Russian).
Suvorov A.K. Topological methods of compilation of cartographic images. Moscow, Academy of Sciences of the USSR, Institute of Geography, 1986, 209 pp (in Russian).
The Library Atlas, 1980, London, Melbourne, Milwaukee, George

Philip.
Tikunov V.S. An algorithm for compilation of typological maps thith the help of computers. — In: Sithesis in cartography, Moscow, Moscow State University Publisher, 1976, p.162–167 (in Russian).
Tikunov V.S. An algorithm for modelling thematical contents of typological maps. — Moscow State University Vestnik, ser. geogr., 1983, N 4, p.78–84 (in Russian).
Tikunov V.S. Method of classification of geographical complexes for compilation of estimation maps. — Moscow State University, Vestnik, ser. geogr., 1985, N 4, p.28–36 (in Russian).
Tikunov V.S. Anamorphated cartographic images: history and methods of their compilation. — Moscow State University, Vestnik, ser. geogr., 1986, N 6, p.45–52 (in Russian).
Tikunov V.S. Topological methods in cartography. — Geodesy and cartography, 1988, N 7, p.57–58 (in Russian).
Tikunov V.S. Anamorphated cartographic images: historical outline and construction techniques. — Cartography (Austral.), 1988, v.17, N 1, p.1–8.
Tikunov V.S. Classification in geography: renaissance or fading? (Experience of formal classifications). — Moscow–Smolensk, Smolensk State University Publisher, 1997, 367 pp (in Russian).
Tikunov V.S. Modelling in cartography. — Moscow., Moscow State University Publisher, 1997, 406 pp (in Russian).
Tikunov V.S., Gusein-Zade S.M. Map Transformations. — Geography Review, 1995, v.9, N 1, p.19–24.
Tikunov V.S., Oreshkina D. "Guided democracy": Russian version or about "some shortcomings" of the electoral culture in regions. — Communication, supplement to the journal "Expert", 2000, N 11-12, p.61–65.
Tikunov V.S., Pipkin J.S. Use of Transformed Images in the Study of Social and Economic Features. — Mapping Sciences and Remote Sensing, 1990, v.27, N 3, p.225–230.
Tikunov V.S., Tsapuk D.A. Stable development of territories: cartographic-information provision. — Moscow-Smulensk, SSU Edition, 1999, 176 pp.
Tikunov V.S., Yudin S.A. Use of transformed cartographic images

in urban construction studies. — Academy of Sciences of the USSR, Izvestiya, ser. geogr., 1987, N 1, p.100–105 (in Russian).
Tikunov V.S., Yudin S.A. Use of transformed cartographic images in urban construction studies. — Mapping sciences and remote sensing, 1987, v.24, N 3, p.202–208.
Tobler W.R. Geographic area and map projections. — Geograhical Review, 1963, v.53, p.59–78.
Tobler W.R. A transformational view of cartography. — Amer. Cartogr., 1979, 6, N 2, p.101–106.
Tobler W.R. Cartograms and cartosplines. — Proceedings of 1976 Workshop on Automated Cartography and Epidemiology. National Center for Helth Statistics. Washington, D.C., 1979, p.53–57.
Tobler W.R. Pseudo–Cartograms. — The American Cartographer, 1986, 13, N 1, p.43–50.
Todorov A.S. Kachestvo zhizni. Kritichesky analiz burzhuaznukh kontseptsii (Quality of living. Critical analysis of bourgeois concepts) Moscow: Progress Publishers, 1980, 222 pp (in Russian).
Trunin Yu.M., Serbenyuk S.N. Maps of accessibility for analysis of properties of the economic–geographical space (on cartographical representation of a surface of negative curvature). — In: Questions of geography, N 77, Mathematics in economic geography. Moscow, Mysl' Publishers, 1968, p.179–185 (in Russian).
UNDP. Human Development Report, 1998. - New York, Oxford University Press, 1998, 228 pp.
Ursul A.D. To the environmentally safe sustainable development. Obshchestv. nauki i sovremennost', 1994, N 4, p.127–135 (in Russian).
Ushmayeva T.M. Indicators of the human health damage in the management of social development. Ekonomika prirodopolzovaniya. Obz. Inf., 1995, N 2, p.40–63 (in Russian).
Uzan L.J. Images d'espaces. — Acta geogr. (Fr.), 1989, N 79, p.12–19.
Vakhrameeva L.A., Bugaevskiy Yu.L. Study of scale–variable projections for social–economic maps. — Izvestiya vuzov, ser. geography and air photography, 1985, N 3, p.95–99 (in Russian).
Vardomskiy L.B., Tikunov V.S. Some approaches to the geographical analysis of foreign trade (with reference to Soviet trade with the Eu-

ropean countries). — Soviet Geography: Rev. and Transl., 1984, 25, N 4, p.253–260.
Vasilevskiy L.I. Anamorphated maps with variable scale and their use in economic geography. — In: New in subjects, contents and methods of compilation of economic maps. Moscow, Moscow branch of the Geographical Society, 1970, p.27–37 (in Russian).
Vorobiev V.V., Snytko V.A. Geographical aspects of sustainable development. Geografiya i prirodnyye resursy, 1996, N 3, p.189–190 (in Russian).
White M.S., Griffin P. Piecewise linear rubber–sheet map transformation. — The American Cartographer, 1985, 12, N 2, p.123–131.
Wonders L.J. The junior atlas of Alberta: introducting new mapping techniques to young students. — Can. Geographer, 1980, v.24, N 3, p.306–311.
Yakshin A.M. et al. Graphic–analitical method in town building researches and planning. Moscow, Strojizdat Publishers, 1979, 204 pp (in Russian).
Yanovsky R.G. Spiritual and moral safety of Russia. Socis, 1995, N 12, p.39–47 (in Russian).
Yargina Z.N. Social bases of town building planning. Moscow, Moscow Architectural Institute, 1982, 132 pp (in Russian).
Zhukov V.I. Reforms in Russia: social results and the development potential. Sotsialnaya rabota: teoriya, tekhnologiya, obrazovaniye, 1996, N 1, p.11–21 (in Russian).
Zhukov V.T., Serbenyuk S.N., Tikunov V.S. Mathematical–cartographical modelling in geography. — Moscow, Mysl' Publisher, 1980, 224 pp.
Zhukov V.T., Tikunov V.S. Use of the Q–modification of the correlation analysis for compilation of maps of industry. — In: Theory and methods of economic–geographical studies. 1977, Moscow, Moscow Branch of the All-Union Geographical Sosiety, p.64–71 (in Russian).
Zhukovskaya V.M., Muchnik I.B. Factor analysis in social-economic studies. Moscow. Statistika Publisher, 1976, 152 pp (in Russian).
Zubarevich N.V., Tikunov V.S. Assessment of regional distinctions of the level of human potential development in Russia. Materialy mezhdunarodnoi konferentsii "GIS dlya ustoichivogo razvitiya territorii", Chast 4, GIS v regionalnykh issledovaniyakh, Yakutsk, Rossiya, 1999,

p.6–20 (in Russian).

Anampphosises as a method of visualization

Sabir M. Gusein–Zade, Vladimir S.Tikunov

Development of a number of Earth sciences, connected with the spatial–temporal analysis, presupposes not only an improvement of methods of representation of geographical phenomena, but also demonstration of their relations and connections with other phenomena, especially in cases when we are analyzing them as systems. Often it is necessary to examine changing over the space characteristics of several phenomena at once. It is much convenient to carry out such an analysis in the case if one of the characteristics is uniformly distributed over the territory and we are regarding all other characteristics against this one as the background. Of cause such a situation is very rear. It arises the idea to create it artificially. For this it is possible to transform the image of the phenomenon taken as the base from usual Euclidean metric of the space into a conditional thematic "space" of the uniformed phenomenon. Under the term "transformation" we understand a transition from the ordinary cartographic image, usually based on the topographic metric of the Earth surface, to another image, based on a metric connected with the phenomenon under consideration. Geographers display growing interest to such transformed images which are called anamorphosises. In other words anamorphosises can be defined as graphical images obtained from the traditional maps, the scale of which is not constant and varies depending on values of some indices, on which they are based.

In English speaking countries instead of anamorphosises terms transformed maps, pseudo-cartograms, cartograms, topological cartograms and so on are used. We prefer to use the term anamorphosis and to call an anamorphation (from the greek word anamorphóō) the process of their creation. It seems that this term reflects the essence of the process, connected with the change of image proportions, more precisely. Besides that let us emphasize that this term is spread in a number of countries, first of all in the Eastern Europe. In Russian scientific language the word "anamorphosis" also has been used for a rather long time. Thus as far back as well-known Russian linguist V.I.Dal' (1881) defined anamorphosis as a hideous but regularly deformed picture, which can be seen in a cut or curved mirror. By the way this definition coincides with one of the methods of creation of anamorphosises, which is used at present. However according to contemporary conception of anamorphotic images the term "hideous" is hardly in its place. A transformation of a cartographic image is produced for theoretical and practical purposes and serves as a tool of the geographical analysis.

Anamorphated images differ from cartoids. The last are abstract graphic images, which do not reflect real spatial relations. Examples of cartoids are: the model of the polarized landscape of Rodoman, typical forms of relief according to Raisz and others. Anamorphosises differ from well–known "mental maps" also.

Among anamorphated images two main classes can be distinguished — linear and area anamorphosises. It is possible to imagine a volumetric anamorphosis, for example in the form of a deformed block-diagrams or relief maps, but they were not realized. Linear anamorphosis often looks like a graph image. Changes of lengths of edges of it permit to change the distances between the regarded units (vertices) depending on values of characteristics taken as the base of the anamorphosis. Examples of linear anamorphosises are: image of system of Moscow subway lines reflecting the accessibility of stations measured in time expenditures, image of distances from shops to a fixed point of a city (for example — from its center), measured in time expenditures again, image of export–import connections of any country with anothers and so on. In all these examples certain spatial relations are preserved, in contrast to statistical graphics consisting of columns of different heights.

The book is constructed in such a way that the reader is offered to pass from simple methods of compilation of anamorphosises to more complicated ones, as if restoring by that the historical chronology of development of this field of science. In the beginning of the book the method of compilation of rather simple linear anamorphosises is shown. For area anamorphosises at first manual methods of compilation are described. Then there are methods of mechanical analogy, of electric simulation and the photographic method. This part ends with a variety of numerical methods of compilation of anamorphosises. The last chapter gives some examples of use of anamorphosises for geographical researches.

Contents

Captions of figures in the book of S.M.Gusein-Zade and V.S.Tikunov

Fig. 1. Net polarised landscape on land and sea. Functional zones and ways of communication: A — for a homogeneous plain inside a continent, B — for a seaside. In the upper raw of the legend for land, in the lower one for sea. 1 – urban historical-architectural reservations; 2 – public service and utilitarian ways of communications; 3 – permanent urban residences and manufacturing industry; 4 – agriculture of high and medium intensity; 5 – natural meadows, pastures, forest industry, hunting, suburban recreational parks; 6 – natural preservations; 7 – recreational residences and touristic roads (after B.B.Rodoman, 1974).
Fig. 2. USA from the New-Yorker's viewpoint.
Fig. 3. USA from the viewpoint of Boston residents.
Fig. 4. World from the viewpoint of Ronald Reagan
Fig. 5. The scheme of Moscow city subway system as an example of a linear anamorphosis.
Fig. 6. Isarithmic map of travel time (in minutes) by car from a fixed point (marked by the cross) in Edmonton, Alberta, Canada (a). Anamorphated image in which the positions of city boundaries and the course of the Saskatchewan River are transformed under the (travel time) metric of Fig. 6a (b). Lines of equal time of travel (in minutes) by a car from a fixed point (marked by the cross) in Edmonton: A — The initial map, in the south-west part of which one can see two isolated islands formed by inversions of the slope; B — the image in the equalarea (isometric) polar projection, where the deformations of the city boundaries and of the Saskatchewan valley are caused by the transformation of the space as a function on the time (after Muller, 1983).
Fig. 7. Linear transformed image based on travel time from Moscow to nearby and remote cities. Accessibility from Moscow to capitals of former Union republics and other big cities of the USSR: 1 — Alma–Ata, 2 — Ashkhabad, 3 — Baku, 4 — Voronezh, 5 — Vilnius, 6 — Vladivostok, 7 — Gorki, 8 — Dyushambe, 9 — Yerevan, 10 — Kiev, 11 — Kishinev, 12 — Krasnoyarsk, 13 — Kuibyshev, 14 — Leningrad,

15 — Minsk, 16 — Novosibirsk, 17 — Odessa, 18 — Riga, 19 — Rostov–on–Don, 20 — Sverdlovsk, 21 — Tallinn, 22 — Tashkent, 23 — Tbilisi, 24 — Frunze, 25 — Kharkov. a — by plane, b — by train (after Trunin, Serbenyuk, 1968).

Fig. 8. Distribution of glass and porcelain shops in Prague, Czech Republic: A — on a conventional map; B — on an anamorphosis compiled in the logarithmic scale of the distance (after Z. Murdych, 1983).

Fig. 9. Transformation of the image of Sweden (A) into the azimuthal logarithmic projection (B), centred at the city of Asby (α) and the migration field, which gives a representation about view points of migrants from the Asby county on other country regions (C).

Fig. 10. Schematic plan of Volgograd, 1990; a — with isolines of access time (in min.) by public transport from the city centre; b — the anamorphosis of the image *a* with the transition to the "time scale". ---- — the city street net; ...... — tram lines with the speed 15,4 km/h; • — the city centre.

Fig. 11. Schematic plan of Volgograd, 1941; a — with isolines of access time (in min.) by public transport from the city centre; b — the anamorphosis of the image *a* with the transition to the "time scale". ---- — the city street net; ...... — tram lines with the speed 15,4 km/h; • — the city centre.

Fig. 12. Linear anamorphosis of change of the social-economic distance from Moscow to the centers of Russian Federation for the railroad passenger communications in 1985 and 2001.

Fig. 13. Linear anamorphosis of change of the social-economic distance from the centers of Russian Federation to Moscow for the railroad passenger communications in 1985 and 2001.

Fig. 14. An anamorphosis based on the absolute volumes of foreign trade connections of the USSR with European countries.

Fig. 15.

Anamorphosises and suplementary maps of export types of the USSR to European countries (the sense of the groups 1–4 is explaned in the text).

Fig. 16.

Anamorphosises and suplementary maps of import types of the USSR to European countries (the sense of the groups 1–4 is explaned in the text).
Fig 17. Functional remoteness of cities of the Central Economic Region from the stadard — Moscow.
Fig. 18. Functional remoteness of cities of the Moscow oblast from the stadard — Moscow.
Fig. 19. The example of transformation of the Martin district (after J.Pravda, 1093).
Fig. 20. Image of the World based on the number of population of countries (according to the atlas The Library..., 1980).
Fig. 21. Image of the World based on the export and import volumes of countries (according to the atlas Philips'..., 1976).
Fig. 22. Image of the World by Voitinski brothers.
Fig. 23. Image of the USA (by P.Muehrcke).
Fig. 24. The image of the New Zealand (after Kelly, Neville, 1985).
Fig. 25. Distinguishing of cities on the anamorphosis of the USA (after C.D.Harrison).
Fig. 26. Representation of the risk of diseases in Scotland; a — number of diseases per 100 000 people, b — sizes of rectangles are proportional to the numbers of population: 100, 2 500, 10 000 (after Forster, 1966).
Fig. 27. Countries of the World from the Medical–geographical atlas compiled in the Geography Institute of the Vrije University (Brussels, Belgium).
Fig. 28. Topological nets: A — the initial one, B — the one topologically transformated with respect to the order, and then by the number of population (after Suvorov, 1986).
Fig. 29. Topological image of the geographical distribution of characterristic nature-endemic diseases of the population. I — America, II — Africa, III — Eurasia, IV — Australia. 1 — boundaries of natural zones; 2 — one characterristic nature-endemic disease coded in a symbolic form: as the area of the square; 3 — topological image of a country or an individual territory with the number of characterristic nature-endemic diseases equal to 7. Number of everywhere spread diseases in the structure of all characterristic nature-endemic diseases of population: 4 — 5 or more; 5 — 4; 6 — 3; 7 — 2; 8 — 1; 9 — there

are no everywhere spread diseases (after Raykh, Maksimova, 1988).
Fig. 30. The initial (A) and the anamorphated (B) images of the ratio of residents of Thailand born in China; a — the ratio of the population (per cent) born in China; b — the number of population in thousands (after Hunter, Meade, 1971).
Fig. 31. The initial (A) and the anamorphated (B) images of the election results in Michigan, a — the number of population in thousands (after Hunter, Meade, 1971).
Fig. 32. States and territories of Australia based on the 1976 census data (after Griffin, 1983).
Fig. 33. Equidemic net on the initial image in the rectangular system of coordinates (after Vasilevskiy, 1970).
Fig. 34. Equidemic net for the territory of Italy (after Vasilevskiy, 1970).
Fig. 35. Anamorphated image of Italy. 1 — the contemporal state border, 2 — the border in 1931, 3 — borders of historical regions (after Vasilevskiy, 1970).
Fig. 36. Equidemic net for azymuth constructions (after Vasilevskiy, 1970).
Fig. 37.
The initial cartographic image of the Alberta province.

Fig. 38.
The image of the Alberta province anamorphated by the number of population.
Fig. 39. Packing of balls: A — the arrangement for which (by a statement of L.Skoda and J.C.Robertson) there is the maximal possible area for each ball; B — the arrangement for which there is the minimal possible area for each ball; C–E — photoes of pieces of the model by the construction of the anamorphosis.
Fig. 40. The initial and the anamorphated images of Canada (after Skoda, Robertson, 1972).
Fig. 41. Model transition from the initial cartographic image to the anamorphated one based on the electric modelling: a — anamorphated image in rectangular coordinate system; b — transformation of rectangular coordinates in the nonhomogeneous space; c — smoothing of

the gradient of the electric field by selecting the tension on the model boundaries; d — smoothing of the gradient of the electric field by changing the boundary configuration.

Fig. 42. Block–diagram of the electric integrator: (SP) — source of power suply; (TD) — the tension divisor; (NI) — null–indicator.

Fig. 43. Anamorphated image of Africa based on the number of population.

Fig. 44. Initial cartographic image of the North Kazakhstan: A — air temperature sums for the period with the average daily temperature over 10^o C; B — productivity of the green mass of maize in centers/hectare; C — productivity of barley in centers/hectare.

Fig. 45. Images anamorphated by the photographic method: A — an anamorphosis based on air temperature sums for the period with the average daily temperature over 10^o C; B — productivity of the green mass of maize on the anamorphosis; C — productivity of barley on the anamorphosis.

Fig. 46. The anamorphosis of the territory of the USA, based on the population of states and compiled with the use of the algorithm of Tobler.

Fig. 47. Arrangement of streets and food shops in Troy.

Fig. 48. Arrangement of food shops in Troy on the background of the anamorphosis compilated on the base of the number of population

Fig. 49. Unifocal projection for the territory of Portugal with respect to Setubal.

Fig. 50. A polyfocal anamorphosis of the World.

Fig. 51. Initial generalised image of the distribution of population over the territory of Czechoslovakia (A) and its anamorphosis (B).

Fig. 52. Anamorphosis of the territory of USA based on the density of population.

Fig. 53. The initial image of San Francisco.

Fig. 54. The anamorphosis of San Francisco, based on the density of distribution of white male population (35–54 years old).

Fig. 55. The anamorphosis of San Francisco, based on the density of distribution of black male population (35–54 years old).

Fig. 56. Anamorphosis of the territory of US compiled with the help of the algorithm (Dougenik, Chrisman and Niemeyer, 1985)

Fig. 72. Anamorphosis of Russia based on the number of population with distinguished regions with prevalence of Russian population.
Fig. 73. The initial map of Russia based on the number of population with distinguished regions with prevalence of Russian population.
Fig. 74. Anamorphosis of the territory of USA compiled on the base of numbers of population of states.
Fig. 75. Anamorphosis of the territory of USA compiled on the base of revenues of states.
Fig. 76. An anamorphosis based on the frequency of mentioning of country's names in the "Nezavisimaya gazeta" newpaper (April 1997-1998).
Fig. 77. An anamorphosis based on the frequency of mentioning of country's names in Internet (Rambler system).
Fig. 78. The anamorphosis of the Atlantic Ocean based on the distribution of ships.
Fig. 79. The pollution of the Ocean, shown on the anamorphosis based on the distribution of ships.
Fig. 80. The anamorphosis of the Atlantic Ocean based on an index of integral accesebility: 1) biggest centres of oil output; 2) regions of intensive fishing.
Fig. 81. Types of morbidity of population of Russia.
Fig. 82. Types of dynamics of indices of total mortality of population of Russia.
Fig. 83. Graphs of dynamics of indices of total mortality of population of Russia.
Fig. 84. Correlations between the distribution of the alive substance in the ocean and its oil layer pollution.
Fig. 85. Distribution of the alive substance in the ocean, kg/m^3, on the anamorphosis, based on the density of ships in the Ocean.
Fig. 86. Correlations between the distribution of the alive substance in the ocean and its oil layer pollution on the anamorphosis.
Fig. 87. Total number of nature-endemic diseases of population of countries. Digits show the number of diseases spread everywhere over the territory of the country. (The anamorphosis of the World is compiled on the base of the numbers of population of the countries.

Coloured pictures.

Fig. 13 of the coloured inset. GDP in current dollars, 1998

Fig. 14 of the coloured inset. Industry value added, 1998

Fig. 15 of the coloured inset. Infant mortality, 2001

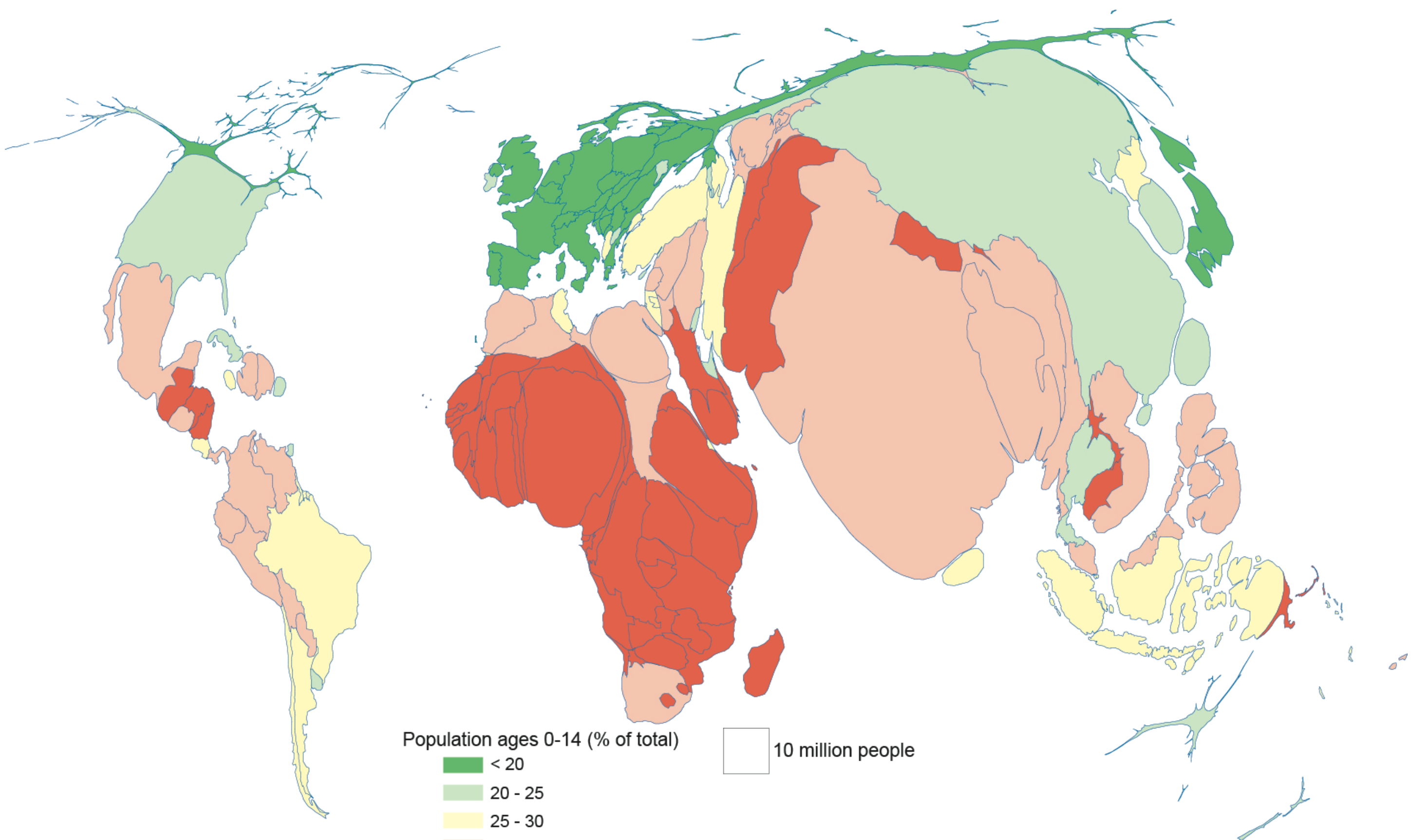
Fig. 16 of the coloured inset. Fertility, 1995-2000

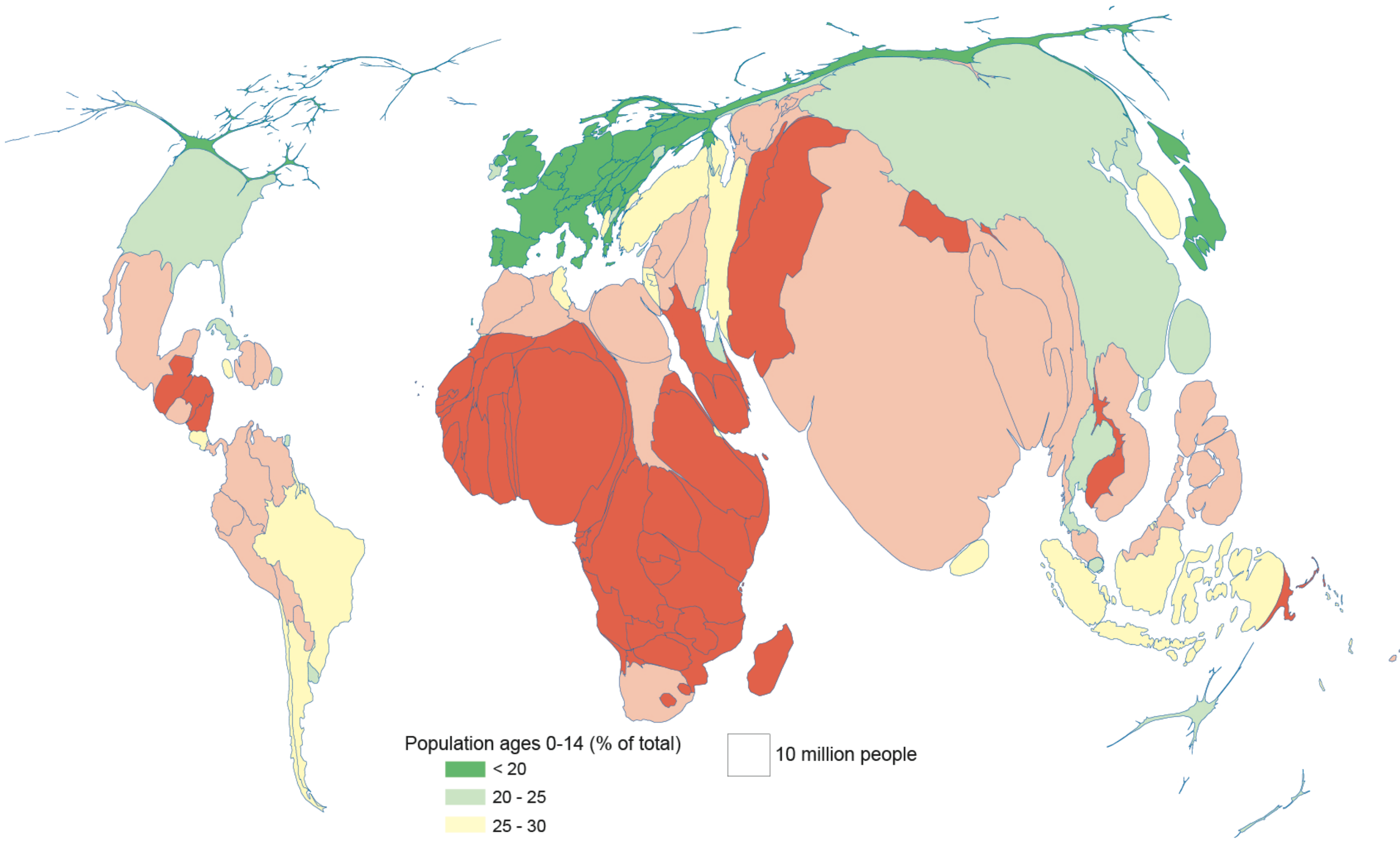
Fig. 17 of the coloured inset. Population using improved water sources, 1999

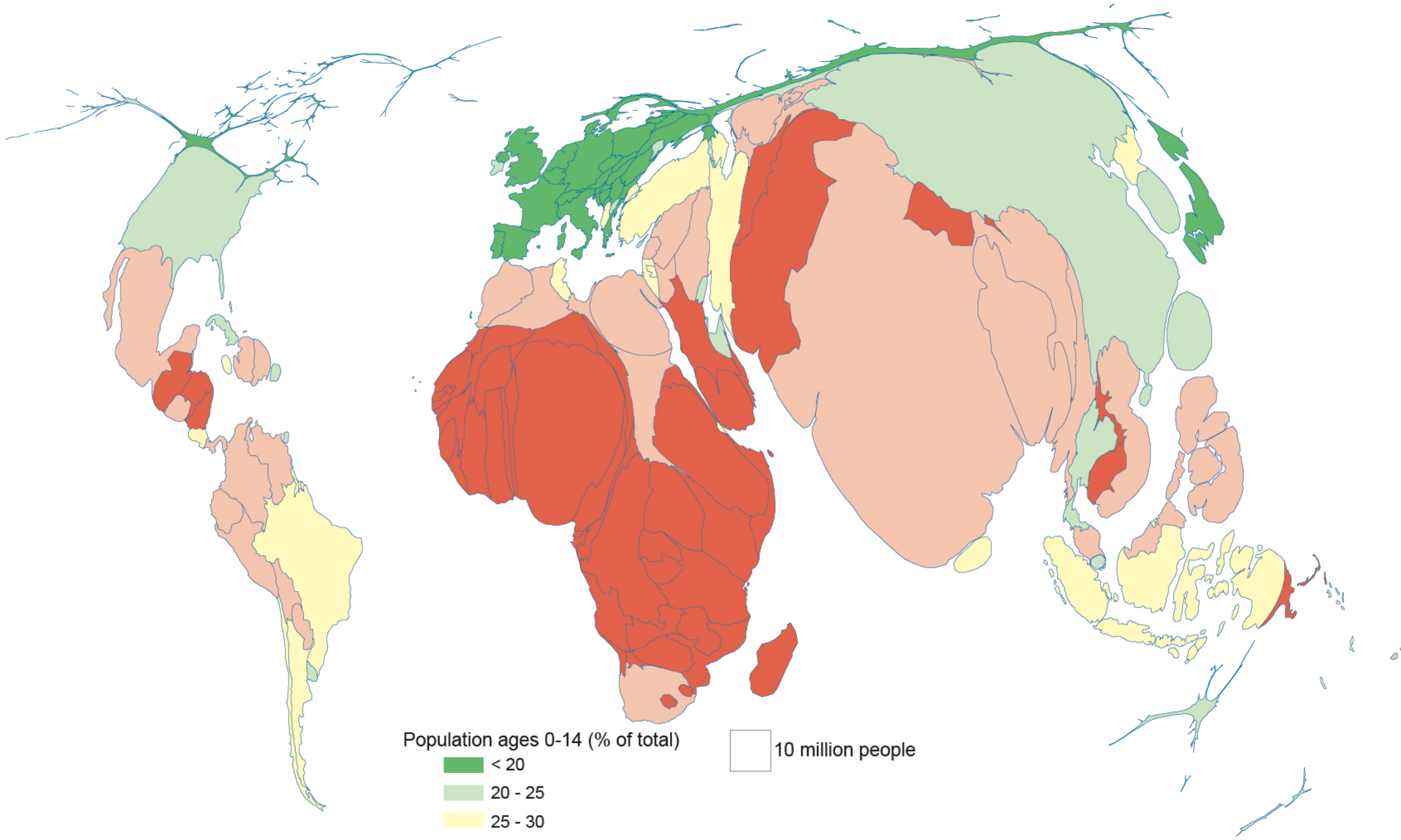
Fig. 18 of the coloured inset. Estimated AIDS deaths, 1999

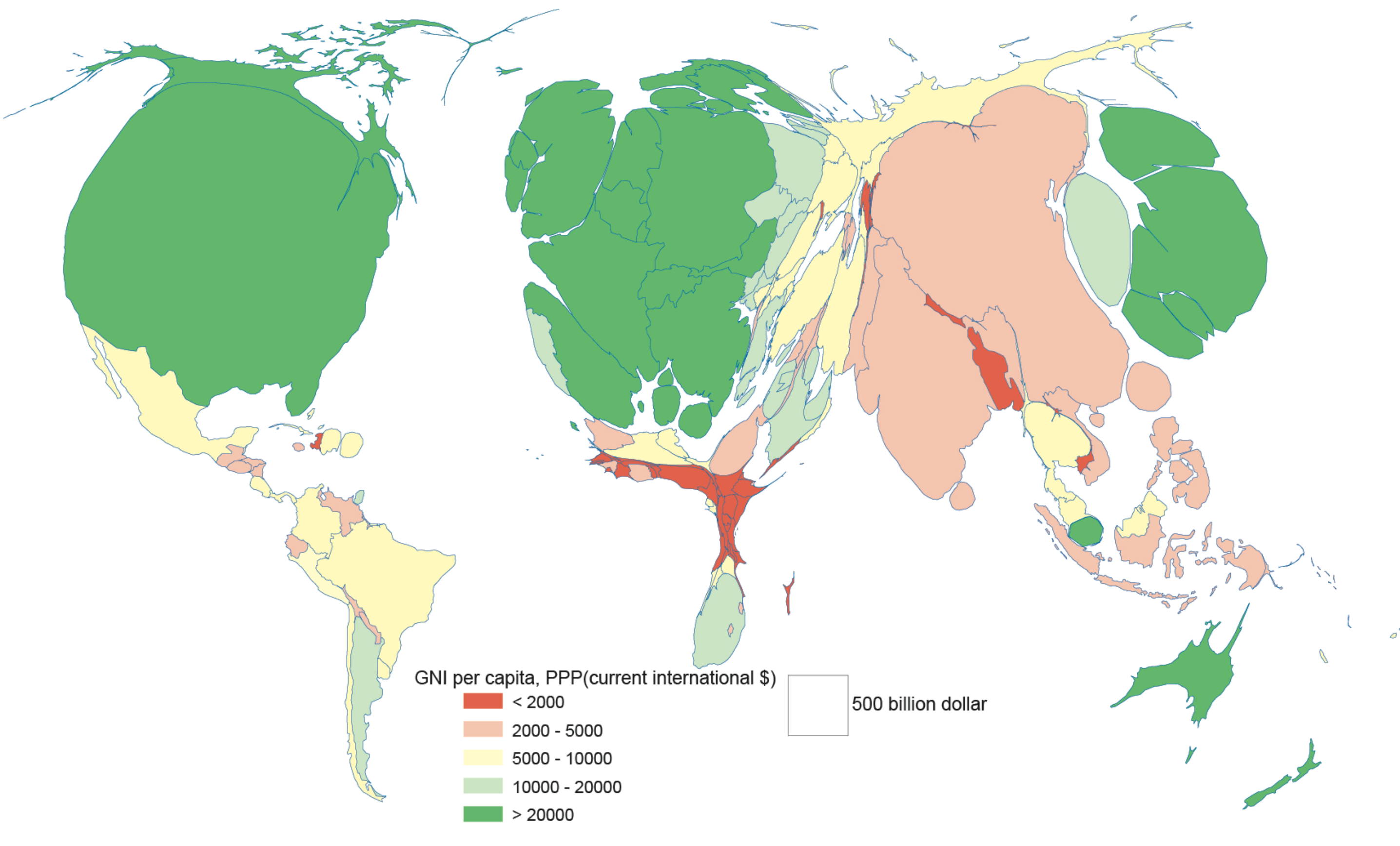
Fig. 19 of the coloured inset. Estimated number of people living with HIV/AIDS, 1999

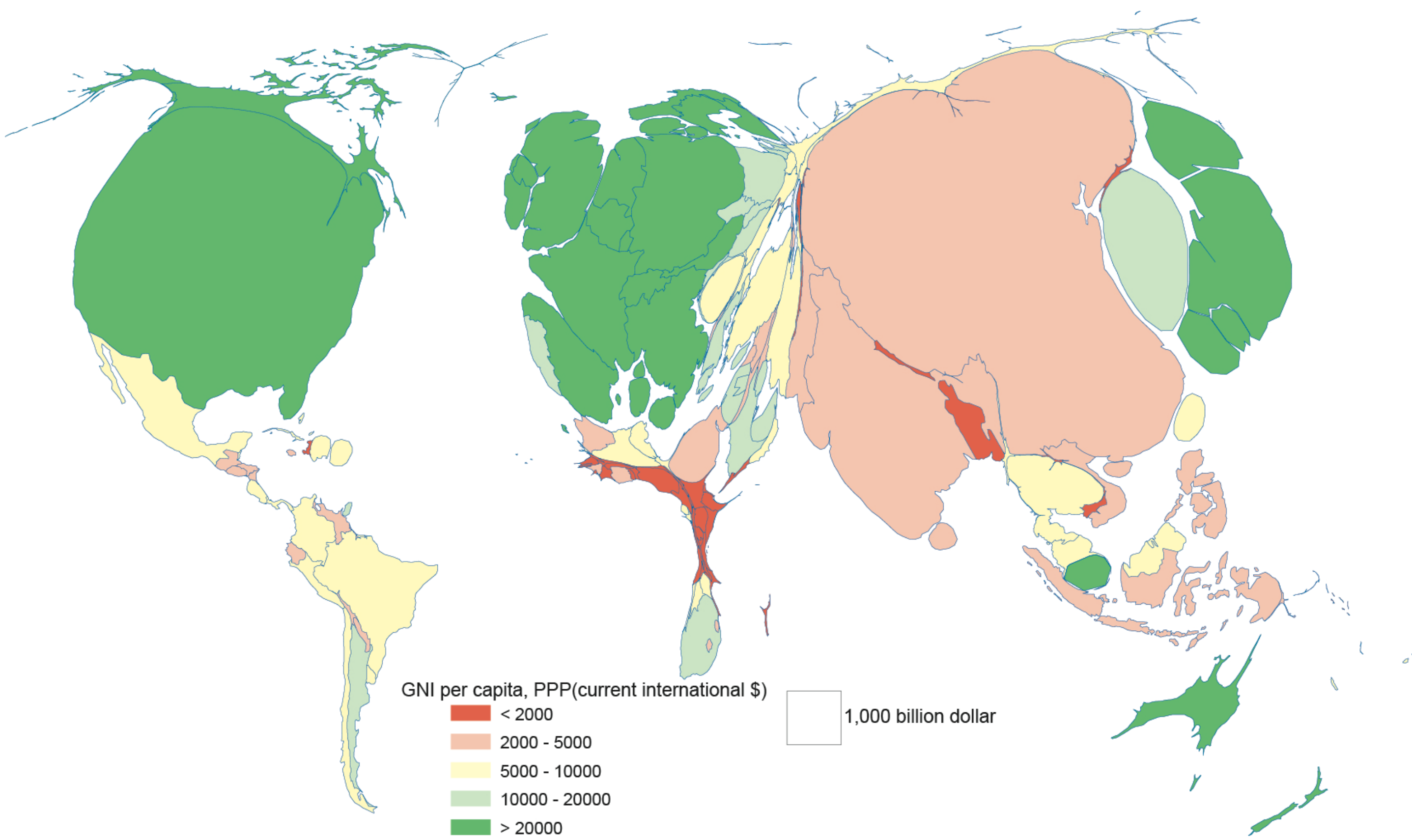
Fig. 20 of the coloured inset. Human development index, 1999

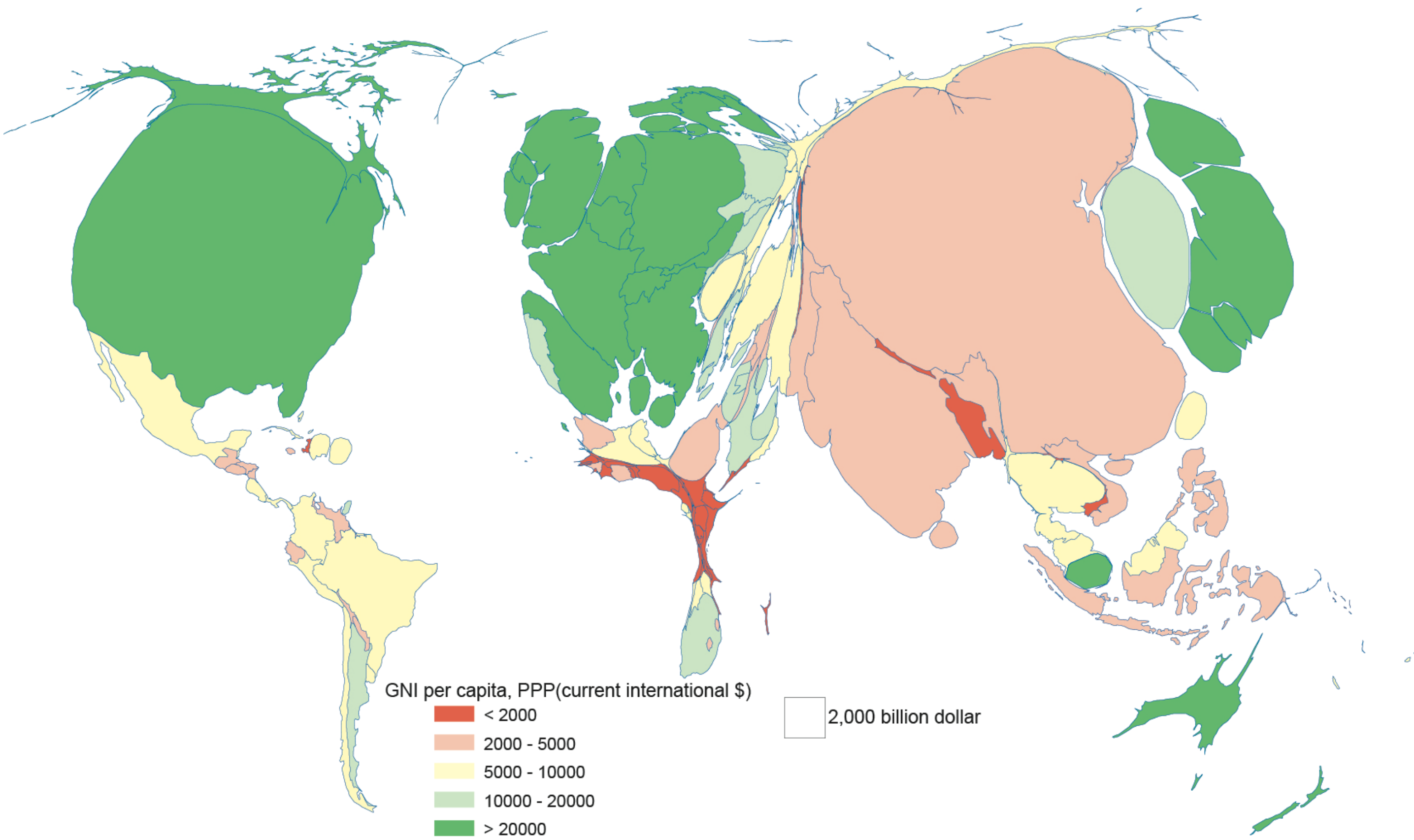

Population ages 0-14 (% of total)
< 20
20 - 25
25 - 30
30 - 40
> 40
10 million people

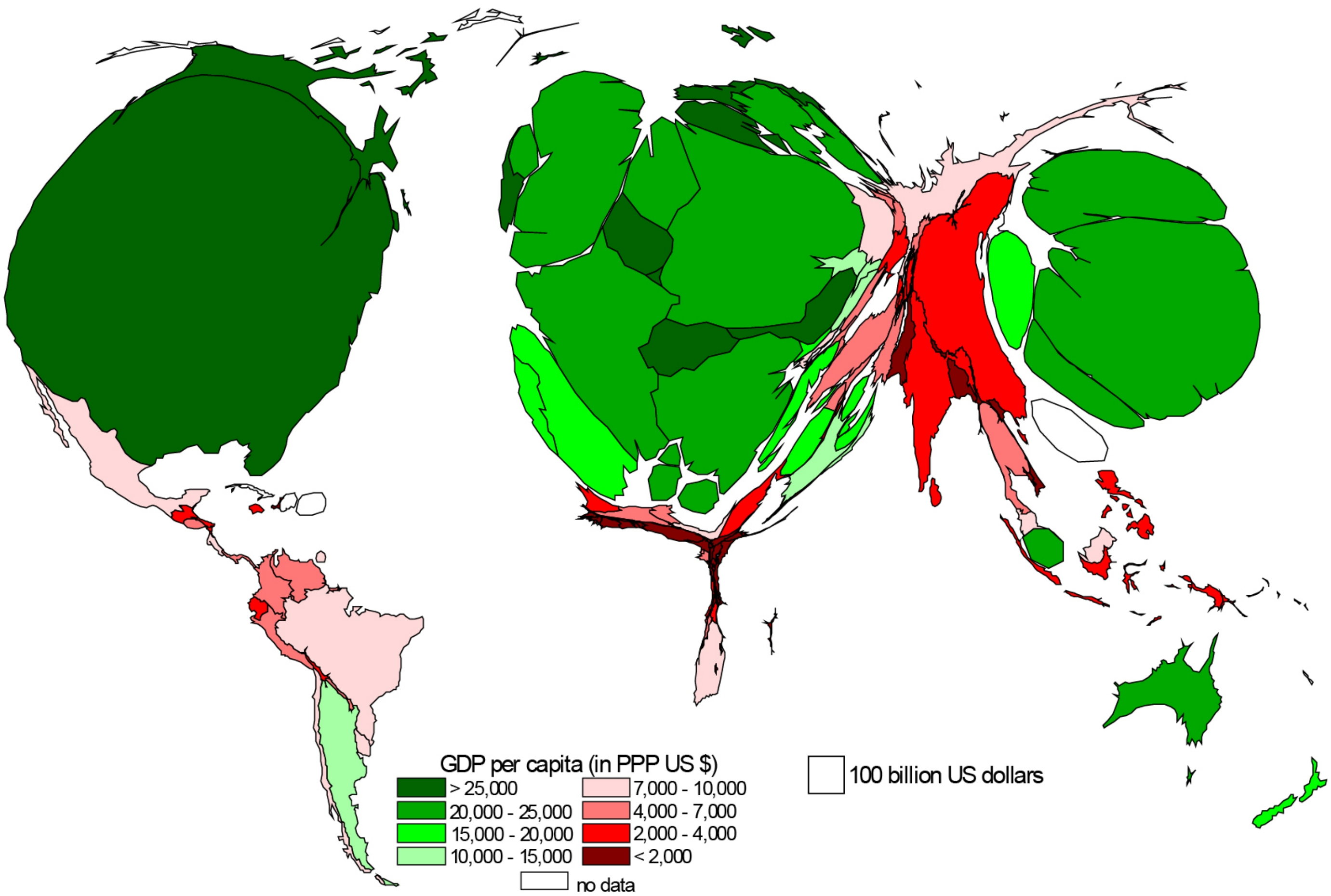

Population ages 0-14 (% of total)
< 20
20 - 25
25 - 30
30 - 40
> 40
10 million people

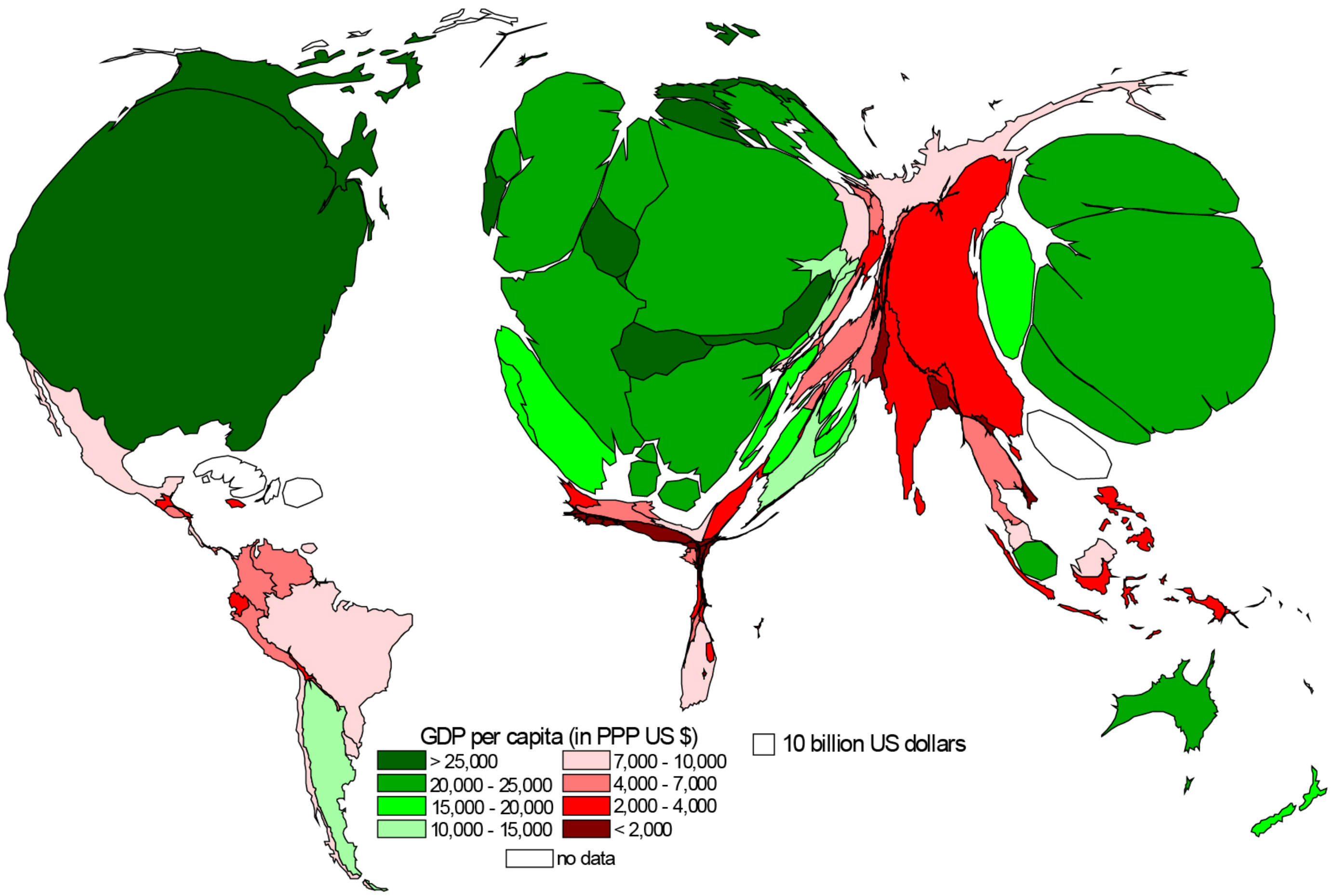

Population ages 0-14 (% of total)
< 20
20 - 25
25 - 30
30 - 40
> 40
10 million people


GNI per capita, PPP(current international $)
< 2000
2000 - 5000
5000 - 10000
10000 - 20000
> 20000
500 billion dollar

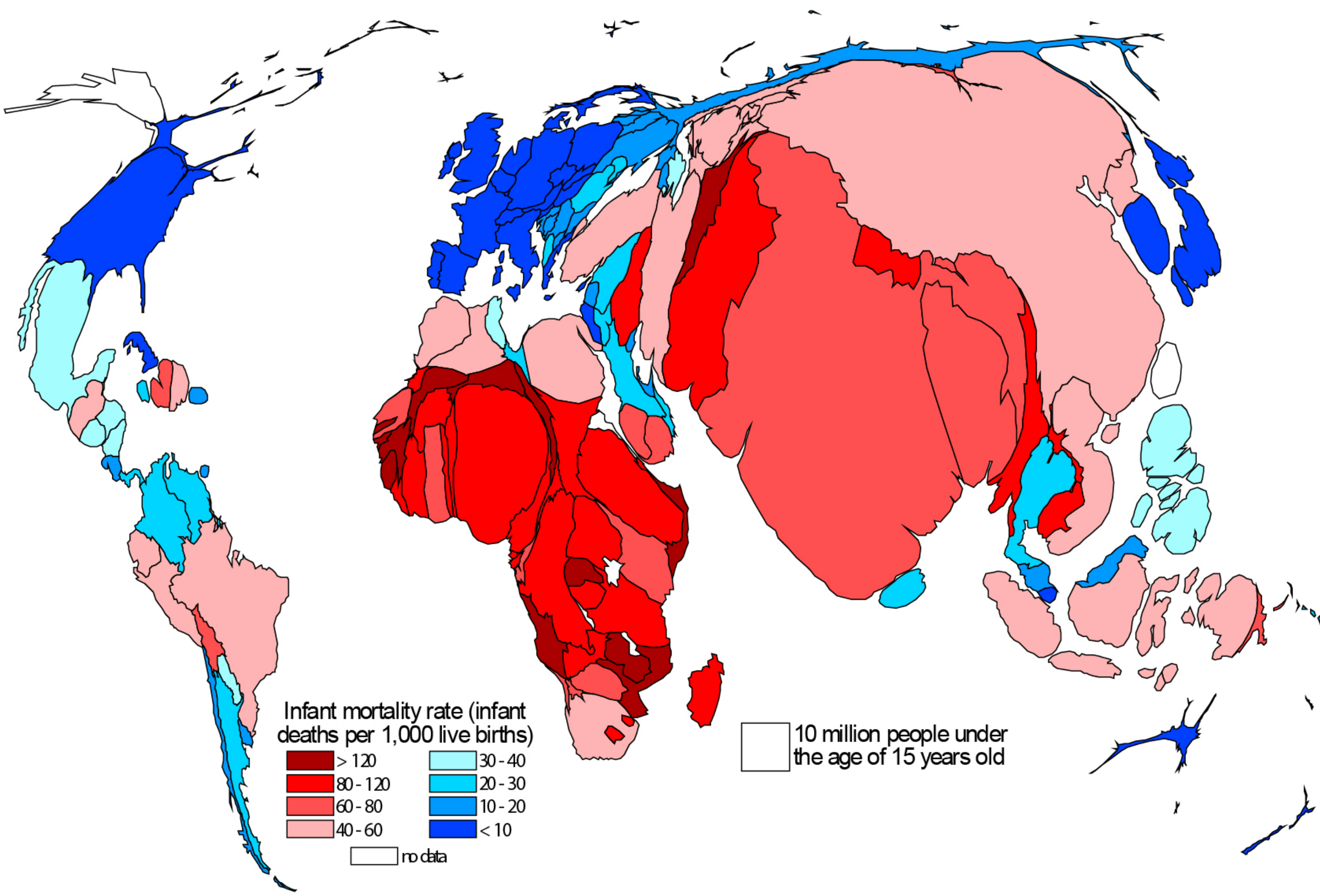

GNI per capita, PPP(current international $)
< 2000
2000 - 5000
5000 - 10000
10000 - 20000
> 20000
1,000 billion dollar


GNI per capita, PPP(current international $)
< 2000
2000 - 5000
5000 - 10000
10000 - 20000
> 20000
2,000 billion dollar

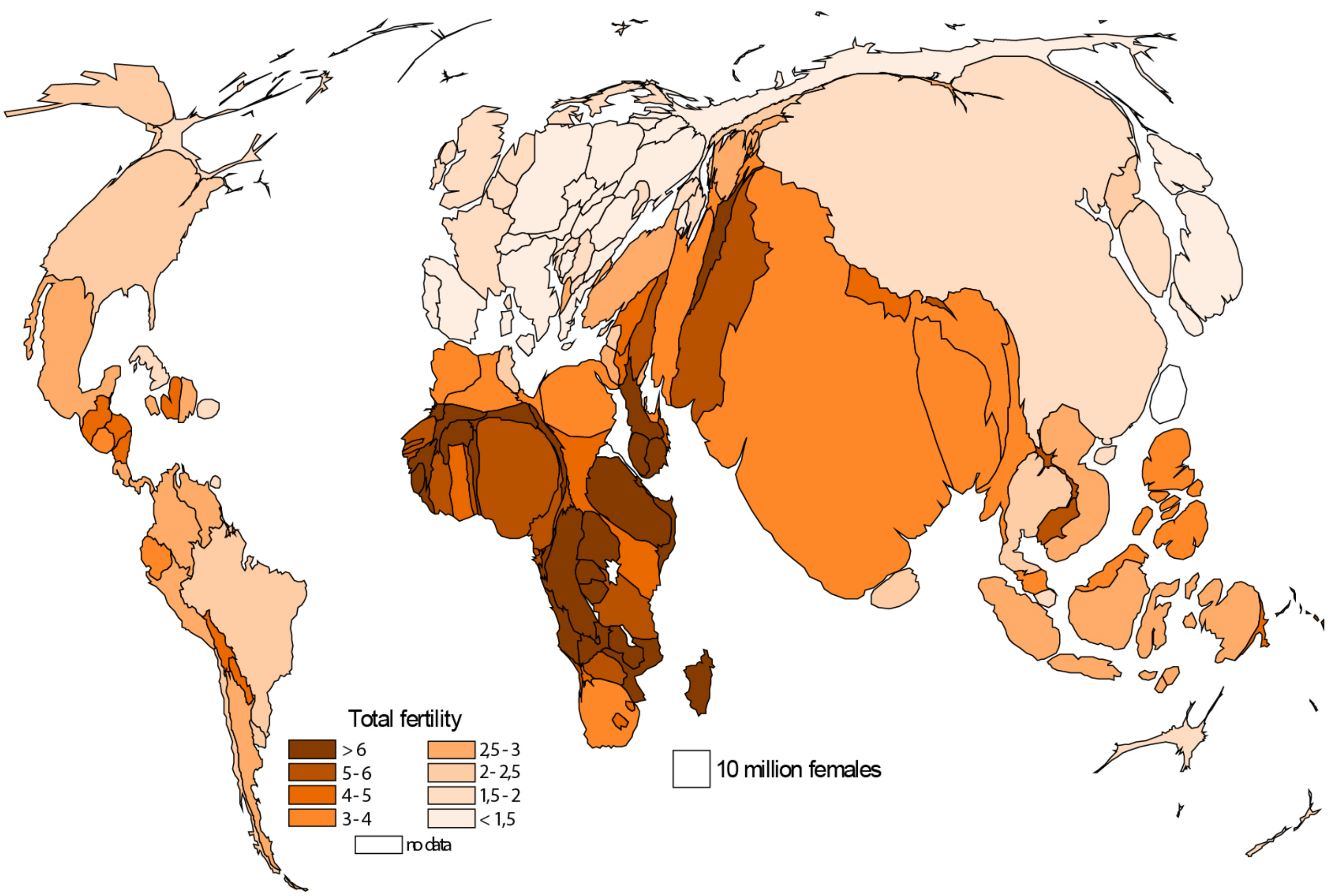

GDP per capita (in PPP US $)
> 25,000
20,000 - 25,000
15,000 - 20,000
10,000 - 15,000
7,000 - 10,000
4,000 - 7,000
2,000 - 4,000
< 2,000
no data
100 billion US dollars



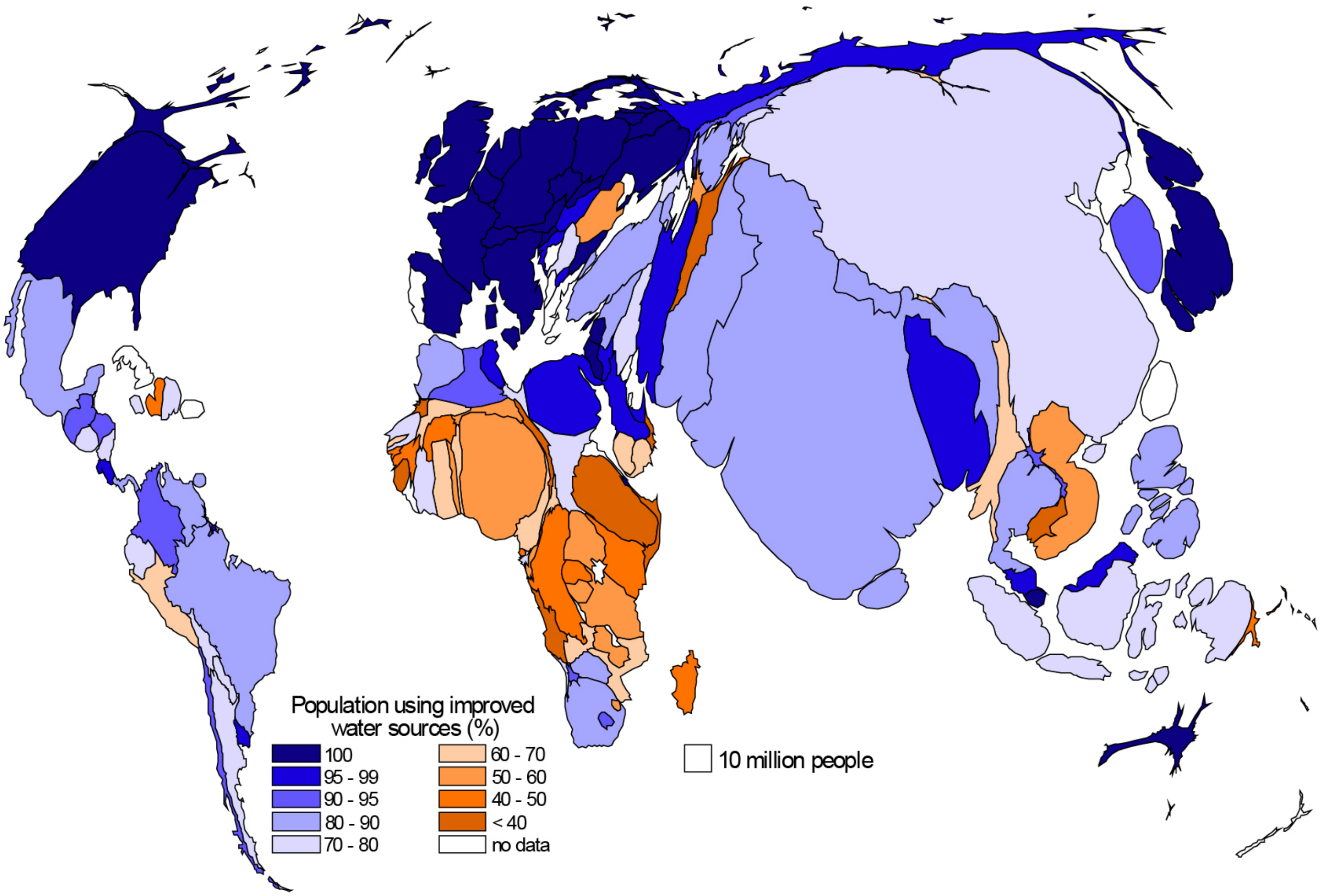
GDP per capita (in PPP US $)
> 25,000
20,000 - 25,000
15,000 - 20,000
10,000 - 15,000
7,000 - 10,000
4,000 - 7,000
2,000 - 4,000
< 2,000
no data
10 billion US dollars

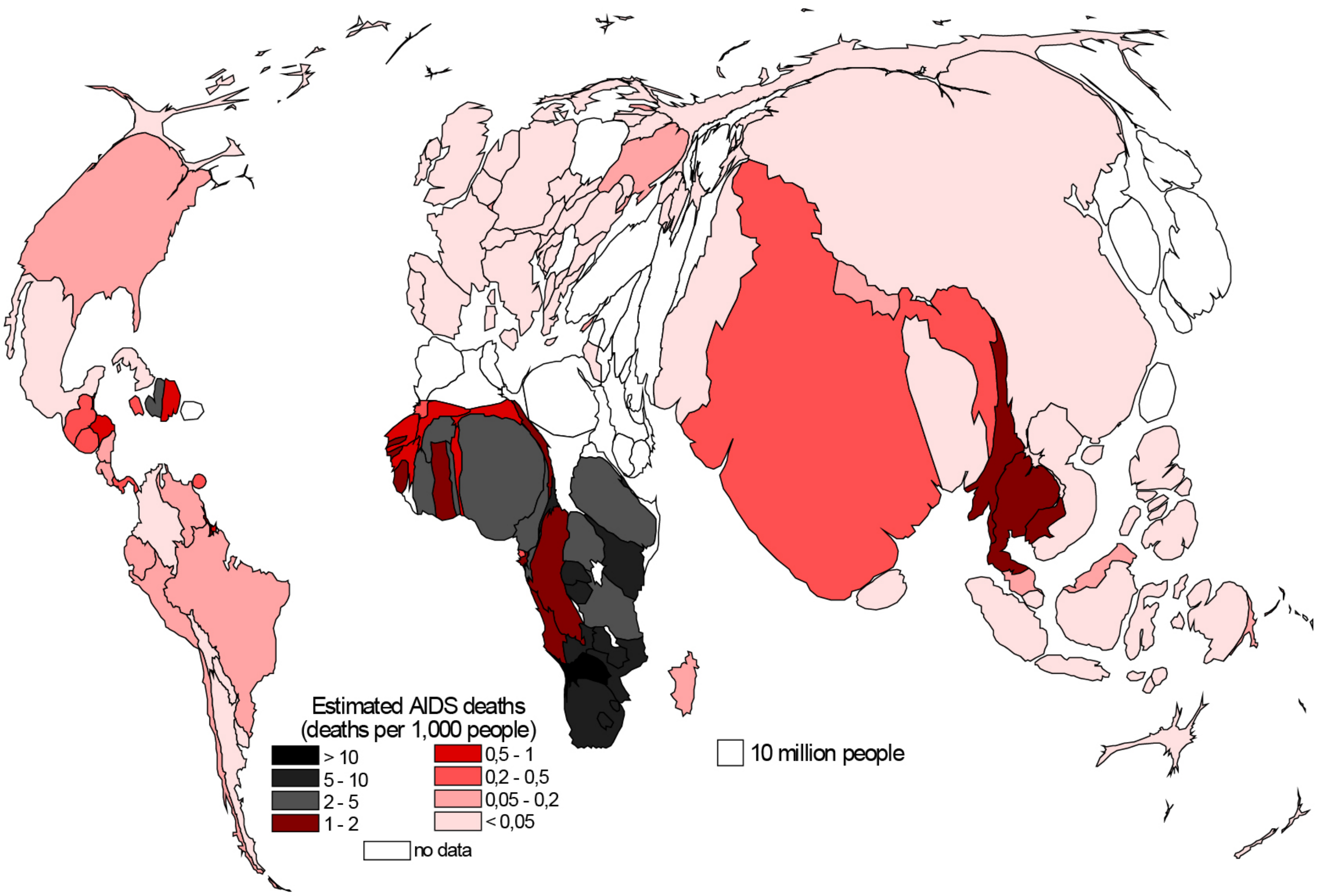


Infant mortality rate (infant deaths per 1,000 live births)
> 120
80 - 120
60 - 80
40 - 60
30 - 40
20 - 30
10 - 20
< 10
no data
10 million people under the age of 15 years old

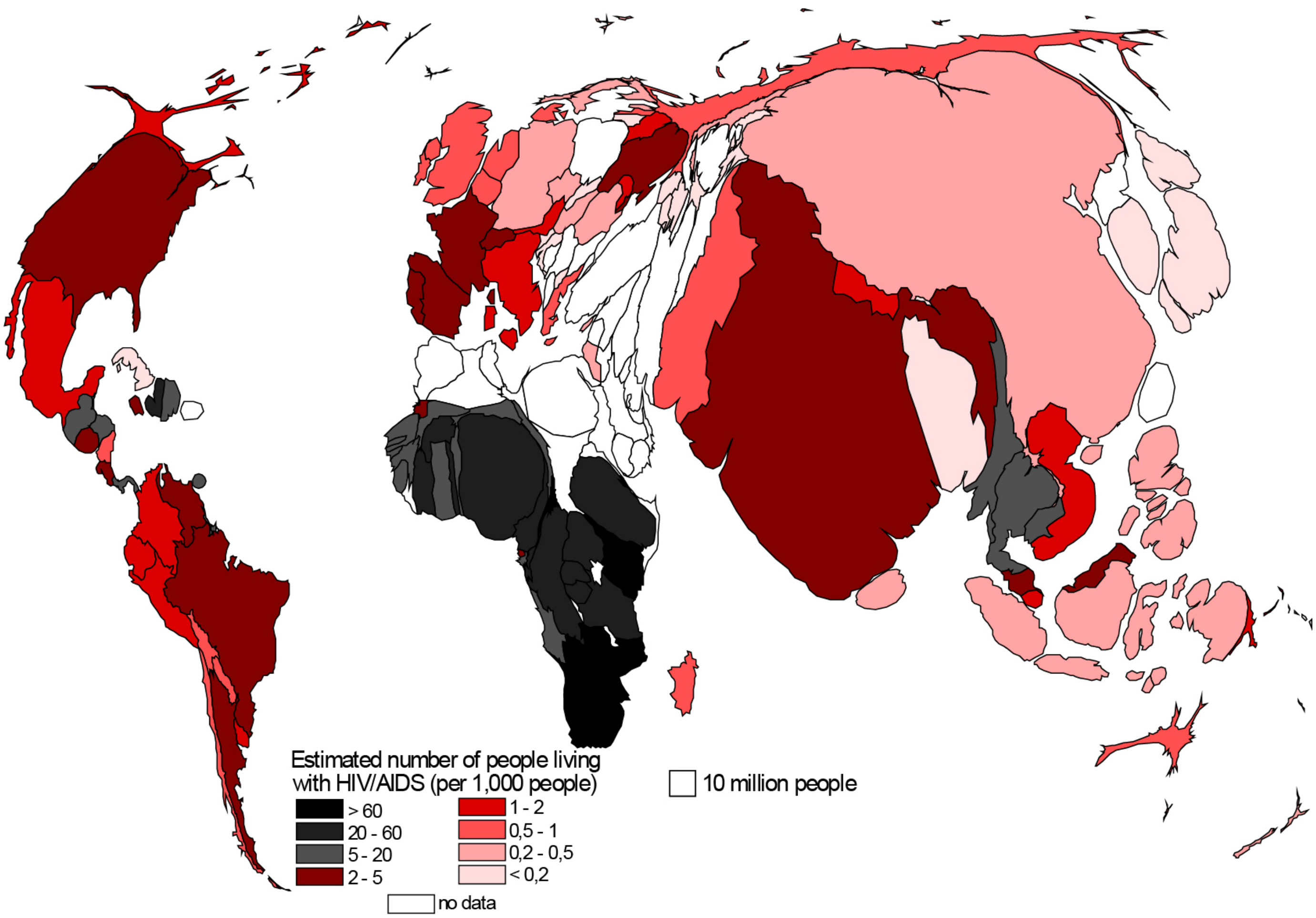


Total fertility
> 6
5 - 6
4 - 5
3 - 4
2,5 - 3
2 - 2,5
1,5 - 2
< 1,5
no data
10 million females

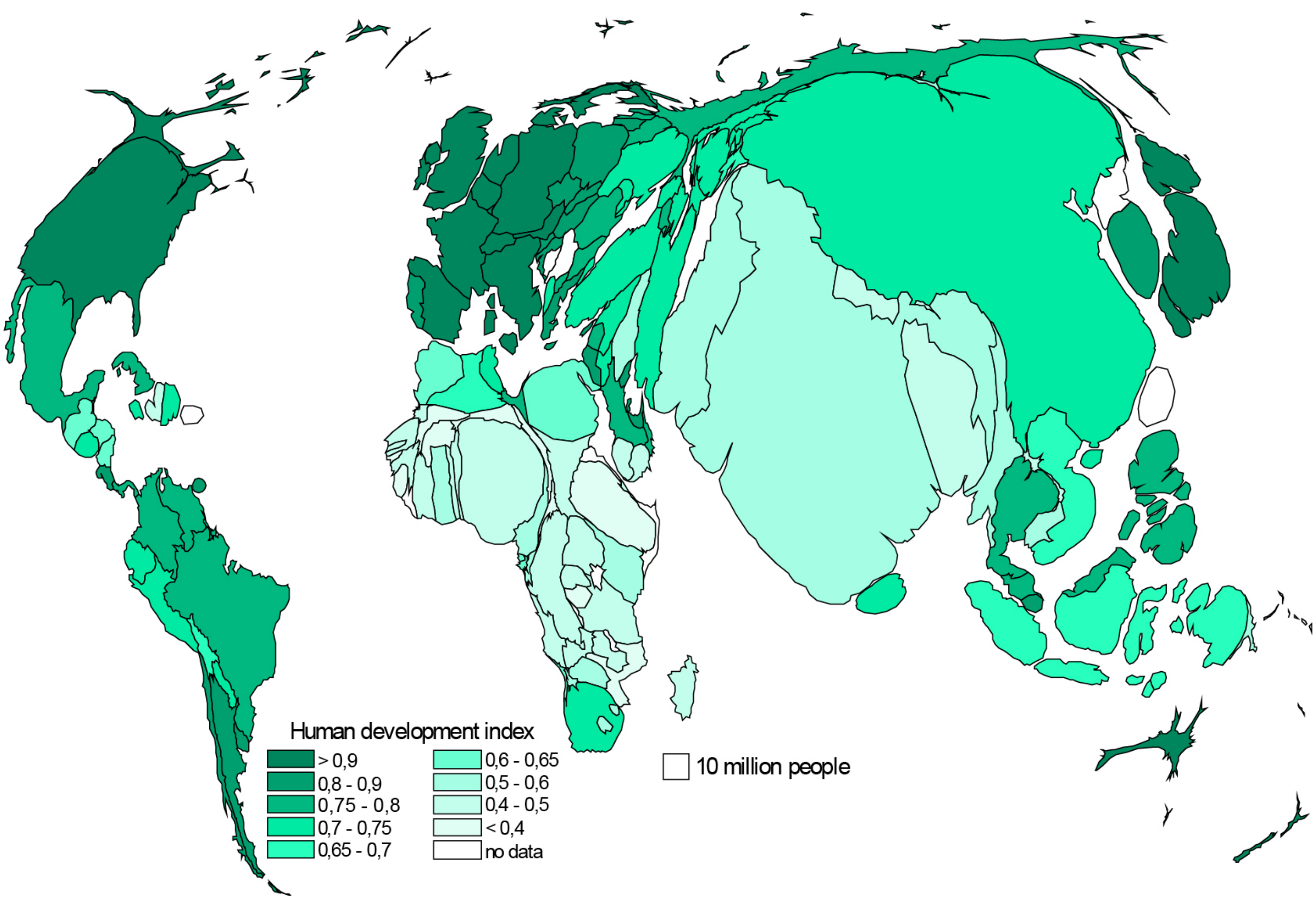

Population using improved water sources (%)
100
95 - 99
90 - 95
80 - 90
70 - 80
60 - 70
50 - 60
40 - 50
< 40
no data
10 million people



Estimated AIDS deaths
(deaths per 1,000 people)
> 10
5 - 10
2 - 5
1 - 2
0,5 - 1
0,2 - 0,5
0,05 - 0,2
< 0,05
no data
10 million people



Estimated number of people living with HIV/AIDS (per 1,000 people)
> 60
20 - 60
5 - 20
2 - 5
1 - 2
0,5 - 1
0,2 - 0,5
< 0,2
no data
10 million people


Human development index
> 0,9
0,8 - 0,9
0,75 - 0,8
0,7 - 0,75
0,65 - 0,7
0,6 - 0,65
0,5 - 0,6
0,4 - 0,5
< 0,4
no data
10 million people

AUTHORS

SABIR M. GUSEIN-ZADE is a Professor of the Department of higher geometry and topology of the Moscow State Lomonosov University. He published more than 180 papers in pure and applied mathematics. In geography his interests concentrate around models of distribution of population and construction of non-traditional cartographic images. He is a member of the Editorial Boards of 8 mathematical journals; in particular he is the Editor-in-chief of the Moscow Mathematical Journal.

VLADIMIR S. TIKUNOV is Professor, Head of the Integrated Mapping Laboratory, Geography Faculty of the Moscow State Lomonosov University, and Director of the Center of the World Data System for Geography. He published over 500 scienti c works, including 14 monographs, texbooks, and manuals in 28 countries of the world and in 14 languages. He has led a number of Russian and international projects. His work has been used in many thematic maps and atlases: National Atlas of Russia (editor-in-chief of Vol. 3), Environmental Atlas of Russia, Atlas of Socio-Economic Development of Russia, Atlas of the Khanty-Mansi Autonomous District Yugra, etc. He is a recipient of the D.N. Anuchin Award for his work in mathematical cartographic modeling. He is also a recipient of the Award in Science and Technology of the Russian Federation Government for the development of environmental and natural-resources atlases of Russia. He was Vice-President and, currently, is Chairman of the commission of the International Cartographic Association and a member of the Commission on Geographic Information Science of the International Geographical Union. He has been a member of the Editorial Boards of 9 Russian and international journals. He lectured at a number of national and international universities. He has been organizing, beginning in 1994, annual international conferences InterCarto-InterGIS "Sustainable Development of Territories: GIS Theory and Practical Experience" that take place both in Russia and abroad.

Institute of Space and Earth Information Science
The Chinese University of Hong Kong

ISBN 978-988-98785-8-0

9 789889 878580 >